Neutrosophic Operational Research

Volume III

Editors:

Prof. Florentin Smarandache

Dr. Mohamed Abdel-Basset

Dr. Victor Chang

pons

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Dedication

Dedicated with love to our parents for the developments of our cognitive minds, ethical standards, and shared do-good values & to our beloved families for the continuous encouragement, love, and support.

Acknowledgment

The book would not have been possible without the support of many people: first, the editors would like to express their appreciation to the advisory board; second, we are very grateful to the contributors; and third, the reviewers for their tremendous time, effort, and service to critically review the various chapters. The help of top leaders of public and private organizations, who inspired, encouraged, and supported the development of this book.

Peer Reviewers

Neutrosophic Operational Research

Volume III

Foreword by John R. Edwards

Preface by the editors
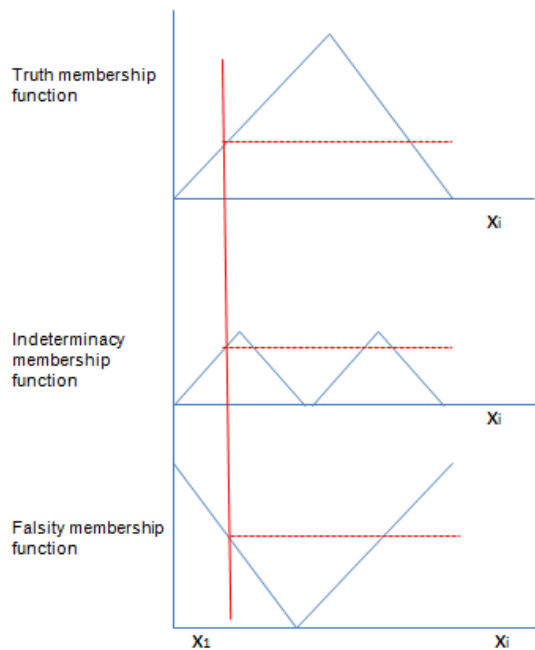
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Contents

Foreword

John R. Edwards

This book is an excellent exposition of the use of Data Envelopment Analysis (DEA) to generate data analytic insights to make evidence-based decisions, to improve productivity, and to manage cost-risk and benefit-opportunity in public and private sectors. The design and the content of the book make it an up-to-date and timely reference for professionals, academics, students, and employees, in particular those involved in strategic and operational decision-making processes to evaluate and prioritize alternatives to boost productivity growth, to optimize the efficiency of resource utilization, and to maximize the effectiveness of outputs and impacts to stakeholders. It is concerned with the alleviation of world changes, including changing demographics, accelerating globalization, rising environmental concerns, evolving societal relationships, growing ethical and governance concern, expanding the impact of technology; some of these changes have impacted negatively the economic growth of private firms, governments, communities, and the whole society.

Preface

Prof. Florentin Smarandache

Dr. Mohamed Abdel-Basset

Dr. Victor Chang

This book treats all kind of data in neutrosophic environment, with real-life applications, approaching topics as logistic center, multi-criteria group decision making, hybrid score-accuracy function, single valued neutrosophic set, single valued neutrosophic number, neutrosophic MOORA, supplier selection, neutrosophic crisp sets, analytic network process, neutrosophic set, multi-criteria decision analysis (MCDM), complex neutrosophic set, interval complex neutrosophic set, interval complex neutrosophic graph of type 1, adjacency matrix, and so on.

In the first chapter *(Neutrosophic Multi-Attribute Group Decision Making Strategy for Logistics Center Location Selection)*, the authors Surapati Pramanik, Shyamal Dalapati and Tapan Kumar Roy use the score and accuracy function and hybrid score accuracy function of single-valued neutrosophic number and ranking strategy for single-valued neutrosophic numbers to model logistics center location selection problem. An illustrative numerical example has been solved to demonstrate the feasibility and applicability of the developed model. As an important and interesting topic in supply chain management, fuzzy set theory has been widely used in logistics center location to improve the reliability and suitability of the logistics center location with respect to the impacts of both qualitative and quantitative factor. However, fuzzy set cannot deal with the indeterminacy involving with the problem. To deal indeterminacy, single-valued neutrosophic set due to Wang et al. [2010] is very helpful. Logistics center location selection having neutrosophic parameters is a multi-attribute making process involving subjectivity, impression and neutrosophicness that can be represented by single-valued neutrosophic sets.

In the second chapter *(A Scientific Decision Framework for Supplier Selection under Neutrosophic Moora Environment)*, the authors Abduallah Gamal, Mahmoud Ismail and Florentin Smarandache present a hybrid model of Neutrosophic-MOORA for supplier selection problems. Making a suitable model for supplier selection is an important issue to ameliorating competitiveness and capability of the organization, factory, project etc.; selecting of the best supplier

selection does not only decrease delays in any organizations, but also gives maximum profit and saving of material costs. Thus, nowadays supplier selection is become competitive global environment for any organization to select the best alternative or taking a decision. From a large number of availability alternative suppliers with dissimilar strengths and weaknesses for different objectives or criteria, requiring important rules or steps for supplier selection. In the recent past, the researchers used various multi criteria decision making (MCDM) methods successfully to solve the problems of supplier selection. In this research, Multi-Objective Optimization based on Ratio Analysis (MOORA) with neutrosophic is applied to solve the real supplier selection problems. The authors selected a real life example to present the solution of problem that how ranking the alternative based on decreasing cost for each alternative and how formulate the problem in steps by Neutrosophic- MOORA technique.

The third chapter *(Foundation of Neutrosophic Crisp Probability Theory)* deals with the application of Neutrosophic Crisp Sets (which is a generalization of Crisp Sets) on the classical probability, from the construction of the Neutrosophic sample space to the Neutrosophic crisp events reaching the definition of Neutrosophic classical probability for these events. Then, the authors Rafif Alhabib, Moustaf Amzherranna, Haitham Farah and A.A. Salama offer some of the properties of this probability, in addition to some important theories related to it. They also come into the definition of conditional probability and Bayes theory according to the Neutrosophic Crisp sets, and eventually offer some important illustrative examples. This is the link between the concept of neutrosophic for classical events and the neutrosophic concept of fuzzy events. These concepts can be applied in computer translators and decision-making theory.

The main objectives of the fourth chapter *(A Novel Methodology Developing an Integrated ANP: A Neutrosophic Model for Supplier Selection)*, by Abduallah Gamal, Mahmoud Ismail and Florentin Smarandache, are to study the Analytic Network Process (ANP) technique in neutrosophic environment, to develop a new method for formulating the problem of Multi-Criteria Decision-Making (MCDM) in network structure, and to present a way of checking and calculating consistency consensus degree of decision makers. The authors use neutrosophic set theory in ANP to overcome the situation when the decision makers might have restricted knowledge or different opinions, and to specify deterministic valuation values to comparison judgments. They formulate each pairwise comparison judgment as a trapezoidal neutrosophic number. The decision makers specify the weight criteria in the problem and compare between each criteria the effect of each criteria against other criteria. In decision making

process, each decision maker should make $\frac{n \times (n-1)}{2}$ relations for n alternatives to obtain a consistent trapezoidal neutrosophic preference relation. In this research, decision makers use judgments to enhance the performance of ANP. The authors introduce a real life example: how to select personal cars according to opinions of decision makers. Through solution of a numerical example, an ANP problem in neutrosophic environment is formulated.

The neutrosophic set theory, proposed by Smarandache, can be used as a general mathematical tool for dealing with indeterminate and inconsistent information. By applying the concept of neutrosophic sets on graph theory, several studies of neutrosophic models have been presented in the literature. In the fifth chapter *(Interval Complex Neutrosophic Graph of Type 1)*, the concept of complex neutrosophic graph of type 1 is extended to interval complex neutrosophic graph of type 1(ICNG1). The authors Said Broumi, Assia Bakali, Mohamed Talea, Florentin Smarandache and V. Venkateswara Rao propose a representation of ICNG1 by adjacency matrix and study some properties related to this new structure.

I

Neutrosophic Multi-Attribute Group Decision Making Strategy for Logistics Center Location Selection

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Abstract

As an important and interesting topic in supply chain management, fuzzy set theory has been widely used in logistics center location to improve the reliability and suitability of the logistics center location with respect to the impacts of both qualitative and quantitative factor. However, fuzzy set cannot deal with the indeterminacy involving with the problem. To deal indeterminacy, single – valued neutrosophic set due to Wang et al. [2010] is very helpful. Logistics center location selection having neutrosophic parameters is a multi-attribute making process involving subjectivity, impression and neutrosophicness that can be represented by single-valued neutrosophic sets. In this paper, we use the score and accuracy function and hybrid score accuracy function of single- valued neutrosophic number and ranking strategy for single- valued neutrosophic numbers to model logistics center location selection problem. Finally, an illustrative numerical example has been solved to demonstrate the feasibility and applicability of the developed model.

Keywords

Logistic center; Multi-criteria group decision making; Hybrid score-accuracy function; Single valued neutrosophic set; Single valued neutrosophic number.

1 Introduction

Logistics systems are essential for economic development and normal functioning of the society. Logistic center location selection problem can be considered as multi-attribute decision making (MADM) problem. Classical strategies [1, 2, 3] for solving MADM problems are capable of deal with crisp numbers that is the ratings and the weights of the attributes are represented by crisp numbers. However, in practical situations, uncertainty plays an important role in MADM problems and decision makers cannot always present the ratings of alternatives by crisp numbers. To deal this situation, fuzzy set (FS) introduced by L. A. Zadeh [4] and intuitionistic fuzzy set (IFS) introduced by K. T. Atanassov [5] are helpful. But it seems that F. Smarandache's book [6] is the most important starting point in the history of dealing with uncertainty characterized by falsity and indeterminacy as independent components. F. Smarandache (1998) grounded the concept neutrosophic set (NS) that is the generalization of FS and IFS. Then, Wang et al. [7] defined single valued neutrosophic set (SVNS) and its various extensions, hybridization and applications [8-72] have been reported in the literature.

Selection of location for the logistics center is based not only on quantitative factors such as costs, distances but also qualitative factors such as environmental impacts and governmental regulations. During the last three decades, several strategies for solving location selection problems have been proposed in the literature. A. Weber [73] studied at first solutions for location selection problems. L. Cooper [74] discussed the calculation aspects of solving certain class of center location problems. L. Cooper [75] also devised a number of heuristic algorithms for solving large locational problems.

Tuzkaya et al. [76] employed the analytic network process (ANP) strategy based on main four factors, namely, benefits, cost, opportunities and risks for locating undesirable facilities. Analytical hierarchy process (AHP), a special case of ANP was employed to solve location problems [77-101]. M. A. Badri [81] combined AHP and goal model approach for international facility location problem. Chang and Chung [82] studied a multi-criteria genetic optimization for distribution network problems.

In fuzzy environment, Chou et al. [83] studied a multi-criteria decision making (MCDM) model for selecting a location for an international tourist hotel. Shen and Yu [84] employed a fuzzy MADM for selection problem of a company. Liang and Wang [85] presented a fuzzy MCDM strategy for facility site selection. Chu [86] proposed facility location selection using fuzzy technique for order preference by similarity to ideal solution (TOPSIS) under group decision. Kahraman et al. [87] presented four different fuzzy MADM strategies for facility

location problem. Farahani et al. [88] presented a comprehensive review on recent development in multi-criteria location problems.

Recently, Pramanik and Dalapati [62] presented generalized neutrosophic soft MADM strategy based on grey relational analysis for logistic center location selection problem. Pramanik et al. [89] studied logistic center location selection strategy based on score and accuracy function and hybrid score accuracy function of single- valued neutrosophic number due to J. Ye [67].

In this paper, we develop a new strategy for multi attribute group decision making (MAGDM) by combining score and accuracy function due to Zhang et al. [71] and hybrid accuracy function due to J. Ye [67]. We also solve a numerical example based on the proposed strategy for logistic center location selection problem in neutrosophic environment.

Remainder of the paper is organized in the following way: Section 2 recalls preliminaries of neutrosophic sets. Section 3 presents attributes for logistic center location selection. Section 4 is devoted to develop MAGDM strategy. Section 5 provides a numerical example of the logistic center location selection problem. In Section 6, we present concluding remarks and future scope of research.

2 Neutrosophic Preliminaries

In this section, we will recall some basic definitions and concepts that are useful to develop the paper.

2.1 Definition: Neutrosophic sets [6]

Let $\mathcal{U}$ be the space of points with generic element in $\mathcal{U}$ denoted by u. A neutrosophic set A in $\mathcal{U}$ is defined as A = {<u, t_A (u), i_A (u), f_A (u)>: u$\in\mathcal{U}$} , where t_A (u): $\mathcal{U} \rightarrow]^- 0, 1^+ [$, i_A (u): $\mathcal{U} \rightarrow]^- 0, 1^+ [$, and f_A (u) : $\mathcal{U} \rightarrow]^- 0, 1^+$ and $^- 0 \leq t_A$ (u) + i_A (u) + f_A (u) $\leq 3^+$.

2.2 Definition: Single valued neutrosophic sets [7]

Let $\mathcal{U}$ be the space of points with generic element in $\mathcal{U}$ denoted by u. A single valued neutrosophic set G in $\mathcal{U}$ is defined as G = {< u, t_G (u), i_G (u), f_G (u) >: u $\in \mathcal{U}$, where t_G (u), i_G (u), f_G (u) $\in$ [0, 1]} and $0 \leq t_G$ (u) + i_G (u) + f_G (u)$\leq$ 3.

2.3 Definition: Single valued neutrosophic number (SVNN) [67]

Let $\mathcal{U}$ be the space of points with generic element in $\mathcal{U}$ denoted by u. A SVNS G in $\mathcal{U}$ is defined as G = {< u, t_G (u), i_G (u), f_G (u) >: u∈ $\mathcal{U}$}, where t_G (u), i_G (u), f_G (u) ∈ [0, 1] for each point u in $\mathcal{U}$ and $0 \leq t_G$ (u) + i_G (u) + f_G (u) ≤3. For a SVNS G in $\mathcal{U}$ the triple < t_G (u), i_G (u), f_G (u) > is called single valued neutrosophic number (SVNN).

2.4 Definition: Complement of a SVNS

The complement of a single valued neutrosophic set G is denoted by G′and defined as

$$G′= \{<p: t_{G′}(u), i_{G′}(u), f_{G′}(u)>, u∈ \mathcal{U} \},$$

where $t_{G′}$(p)= {1} - t_G (u), $i_{G′}$(u) = {1} − i_G(u), $f_{G′}$(u) = {1} - f_G(u).

For two SVNSs G_1 and G_2 in $\mathcal{U}$, G_1 is contained in G_2, i.e. $G_1 \subseteq G_2$, if and only if t_{G_1} (u) $\leq t_{G_2}$ (u), i_{G_1} (u) $\geq i_{G_2}$ (u), f_{G_1} (u) $\geq f_{G_2}$ (u) for any u in $\mathcal{U}$.

Two SVNSs G_1 and G_2 are equal, written as $G_1 = G_2$, if and only if $G_1 \subseteq G_2$ and $G_2 \subseteq G_1$.

2.5 Conversion between linguistic variables and single valued neutrosophic numbers

A linguistic variable simply presents values that are represented by words or sentences in natural or artificial languages. Importance of the decision makers are differential in the decision making process. Ratings of criteria are expressed using linguistic variables such as very unimportant (VUI), unimportant (UI), medium (M), important (I), very important (VI), etc. Linguistic variables are transformed into single valued neutrosophic numbers as presented in Table- 1.

2.6 Definition: Score function and accuracy function [71]

Assume that x = (t_1, i_1, f_1) be a SVNN. Score function and accuracy function of 'x' are expressed as follows:

$$s(x) = t_1 + 1 - i_1 + 1 - f_1 = 2 + t_1 - i_1 - f_1 \tag{1}$$

$$ac(x) = t_1 - f_1 \tag{2}$$

Here, $s(x)$ and $ac(x)$ represent the score and accuracy function of 'x' respectively.

2.7 Definition 6 [71]

Let 'x' and 'y' are two SVNNs. Then, the ranking strategy can be defined as follows:

(1) If $s(x) > s(y)$, then $x > y$;

(2) If $s(x) = s(y)$ and $ac(x) \geq ac(y)$, then $x \geq y$;

(3) If $s(x) = s(y)$ and $ac(x) = ac(y)$, then x is equal to y, and denoted by $x \sim y$.

3 Selection criteria for logistics center

In order to perform a complete assessment of logistics center location problem as a multiple criteria decision making problem, we choose six criteria adopted from the study [90] namely, cost (C_1), distance to suppliers (C_2), distance to customers (C_3), conformance to governmental regulations and laws (C_4), quality of service (C_5) and environmental impact (C_6).

4 MAGDM strategy based on a new hybrid score accuracy function under SVNNs

The following notations are adopted in the paper.

$J = \{J_1, J_2 \ldots, J_n\}\,(n \geq 2)$ is the set of logistics centers;

$C = \{C_1, C_2, \ldots, C_\rho\}\,(\rho \geq 2)$ is the set of criteria;

$D = \{D_1, D_2, \ldots, D_m\}\,(m \geq 2)$ is the set of decision makers or experts.

The weights of the decision- makers are completely unknown and the weights of the criteria are incompletely known in the group decision making problem. We now present a new hybrid score – accuracy function by combining score and accuracy function due to Zhang et al. [71] and hybrid accuracy function due to Ye [67] for MCDM problem with unknown weights under single-valued neutrosophic environment. MCGDM strategy is presented using the following steps.

Step – 1 Construction of the decision matrix

In the group decision process, if m decision makers or experts are required in the evaluation process, then the r-th (r = 1, 2,…, m) decision maker can provide the evaluation information of the alternative J_i (i =1, ..., n) on the criterion C_j (j = 1, ..., ρ) in linguistic terms that can be expressed by the SVNN (see Table 1). A MCGDM problem can be expressed by the following decision matrix:

$$
\mathbf{A_r} = (x_{ij}^s)_{n \times \rho} =
\begin{pmatrix}
 & C_1 & C_2 & ...... & C_\rho \\
J_1 & x_{11}^r & x_{12}^r ... & & x_{1\rho}^r \\
J_2 & x_{21}^r & x_{22}^r & & x_{2\rho}^r \\
. & . & & ... & . \\
J_n & x_{n1}^r & x_{n2}^r & ... & .x_{n\rho}^r
\end{pmatrix}
\tag{3}
$$

Here $x_{ij}^r = (t_{ij}^r, i_{ij}^r, f_{ij}^r)$ and $0 \le t_i^r(C_j) + i_i^r(C_j) + f_i^r(C_j) \le 3$

$t_i^r(C_j), i_i^r(C_j), f_i^r(C_j) \in [0, 1]$.

For r = 1, 2, ..., m, j =1, 2, ... ρ, i = 1, 2, ...n.

Step – 2 Calculate hybrid score – accuracy matrix

The score – accuracy matrix in hybridization form $\Theta^r = (\zeta_{ij}^r)_{n \times \rho}$ (r = 1, 2……, m; i = 1, 2, ..., n; j = 1, 2, …, ρ) can be obtained from the decision matrix $M_r = (x_{ij}^r)_{n \times \rho}$. The hybrid score-accuracy matrix Θ^r is expressed as

$$
\Theta^r = (\zeta_{ij}^r)_{n \times \rho} =
\begin{pmatrix}
 & C_1 & C_2 & ..... & C_\rho \\
J_1 & \zeta_{11}^r & \zeta_{12}^r ... & & \zeta_{1\rho}^r \\
J_2 & \zeta_{21}^r & \zeta_{22}^r & & \zeta_{2\rho}^r \\
. & . & & ... & . \\
J_n & \zeta_{n1}^r & \zeta_{n2}^r & ... & .\zeta_{n\rho}^r
\end{pmatrix}
\tag{4}
$$

$$
\zeta_{ij}^r = \alpha\ (2 + t_{ij}^r - i_{ij}^r - f_{ij}^r) + (1 - \alpha)\ (t_{ij}^r - f_{ij}^r)
\tag{5}
$$

Here $\alpha \in [0, 1]$. When $\alpha = 1$, the equation (5) reduces to equation (1) and when $\alpha = 0$, the equation (5) reduces to equation (2).

Step – 3 Calculate the average matrix

From the obtained hybrid-score–accuracy matrix, the average matrix

$\Theta^{*}= (\zeta_{ij}^{*})\,n\times\rho$ (r =1, 2, ..., m; i = 1, 2, ..., n; j = 1, 2, ..., ρ) is

expressed by

$$\Theta^{*} = (\zeta_{ij}^{*})\,n\times\rho = \begin{pmatrix} & C_1 & C_2 & \dots\dots & C_\rho \\ J_1 & \zeta_{11}^{*} & \zeta_{12}^{*}\dots & & \zeta_{1\rho}^{*} \\ J_2 & \zeta_{21}^{*} & \zeta_{22}^{*} & & \zeta_{2\rho}^{*} \\ . & . & & \dots & . \\ J_n & \zeta_{n1}^{*} & \zeta_{n2}^{*}\dots & & \zeta_{n\rho}^{*} \end{pmatrix} \tag{6}$$

Here $\zeta_{ij}^{*} = \dfrac{1}{m}\ \sum_{r=1}^{m}\,(\zeta_{ij}^{r})$ \hfill (7)

Collective correlation co-efficient between Θ^{r} (r = 1, 2, ..., m) and Θ^{*} due to Ye (nd.) is presented as follows:

$$\chi_{r} = \sum_{i=1}^{n}\ \ \frac{\sum_{j=1}^{\rho}\zeta_{ij}^{r}\zeta_{ij}^{*}}{\sqrt{\sum_{j=1}^{\rho}(\zeta_{ij}^{r})^{2}}\ \sqrt{\sum_{j=1}^{\rho}(\zeta_{ij}^{*})^{2}}} \tag{8}$$

Step – 4 Determine decision makers' weights

In order to deal with personal biases of decision makers, Ye [67]) suggested to assign very low weights to the false or biased opinions. Weight model due to Ye [67] can be written as follows:

$$\Im_{r} = \frac{\chi_{r}}{\sum_{r=1}^{m}\chi_{r}}\ \ \ , 0\leq\Im_{r}\leq 1, \sum_{r=1}^{m}\Im_{r} = 1 \text{ for r} = 1, 2, \dots, \text{m.} \tag{9}$$

Step – 5 Calculate collective hybrid score – accuracy matrix

For the weight vector $\Im = (\Im_{1}, \Im_{2}, \dots, \Im_{m})^{T}$ of decision makers obtained from equation (6), we accumulate all individual hybrid score – accuracy matrix

$\Theta^{\mathrm{r}} = (\zeta_{ij}^{\mathrm{r}})_{n \times \rho}$ (r = 1, 2,..., m; i = 1, 2, ..., n; j = 1, 2, ..., ρ) into a collective hybrid

score accuracy matrix

$$\Theta = (\zeta_{ij})_{n \times \rho} = \begin{pmatrix} & C_1 & C_2 & \ldots \ldots & C_\rho \\ J_1 & \zeta_{11} & \zeta_{12} \cdots & & \zeta_{1\rho} \\ J_2 & \zeta_{21} & \zeta_{22} & & \zeta_{2\rho} \\ . & . & & \cdots & . \\ J_n & \zeta_{n1} & \zeta_{n1} \cdots & \cdot \zeta_{n1} \end{pmatrix} \tag{10}$$

Here $\zeta_{ij} = \Sigma_{r=1}^{m} \Im_r \zeta_{ij}^{r}$ (11)

Step – 6 Weight model for criteria

To deal decision making problem with partially known weights of the criteria, the following optimization model due to Ye [67] *is employed.*

$$\text{Max } \omega = \frac{1}{n} \Sigma_{i=1}^{n} \Sigma_{j=1}^{\rho} \omega_j \zeta_{ij} \tag{12}$$

Subject to

$$\Sigma_{j=1}^{\rho} \omega_j = 1$$

$$\omega_j > 0$$

Solving the linear programming problem (8), the weight vector of the criteria $\omega = (\omega_1, \omega_2, ..., \omega_n)^{\mathrm{T}}$ can be easily determined.

Step – 7 Rank the alternatives

In order to ranking alternatives, all values can be summed in each row of the collective hybrid score-accuracy matrix corresponding to the criteria weights by the overall weight hybrid score-accuracy value of each alternative J_i (i = 1, 2, . . . , n):

$$\eta(J_i) = \Sigma_{j=1}^{\rho} \omega_j \zeta_{ij} \tag{13}$$

Based on the values of $\eta(J_i)$ *(i = 1, 2, ..., n),* we can rank alternatives J_i (i = 1, 2, ..., n) in descending order and choose the best alternative.
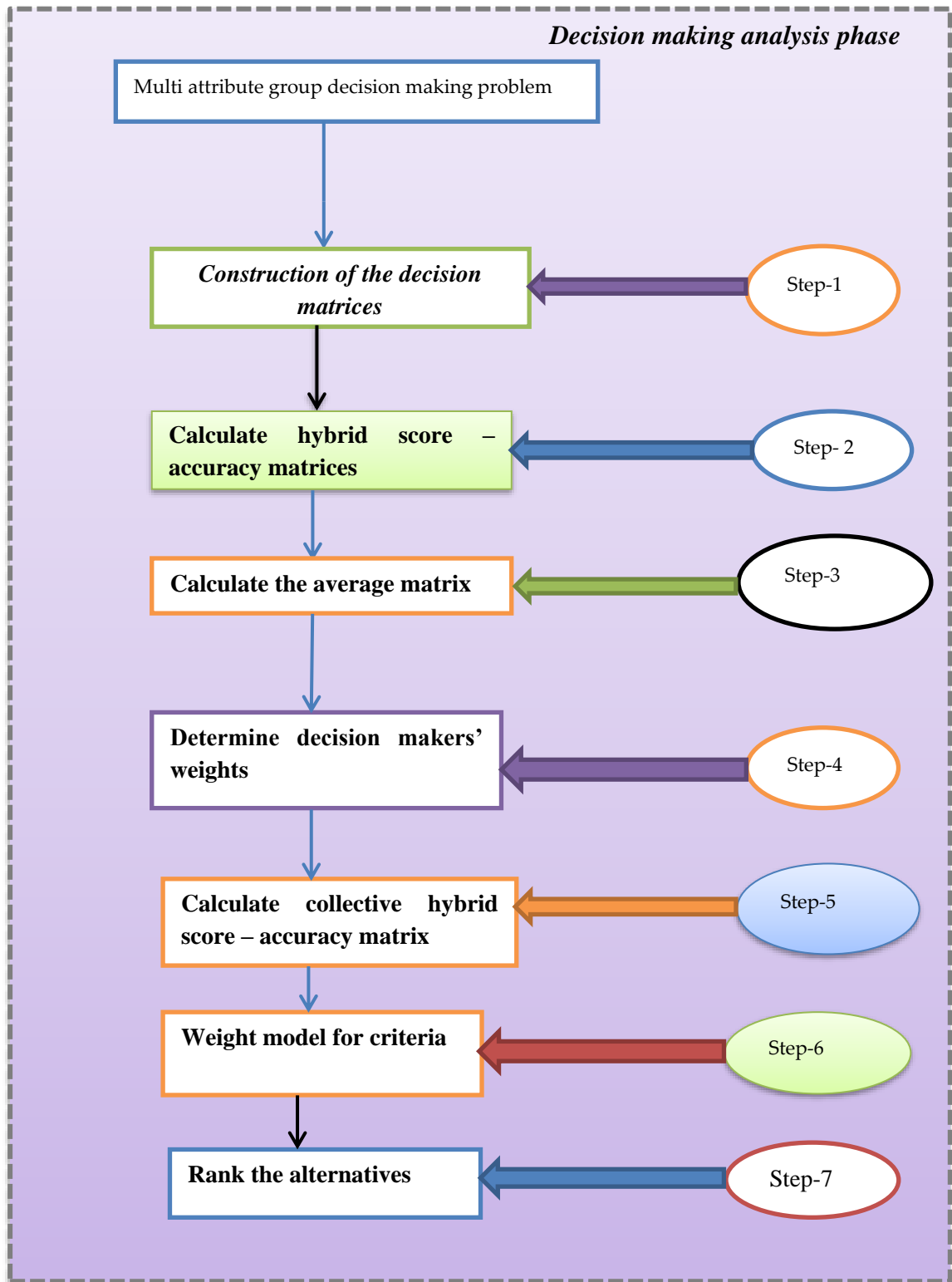
Figure 1. Steps of proposed MAGDM strategy

5 Example of the Logistics Center Location

Assume that a new modern logistic center is required in a town. There are four location J_1, J_2, J_3, J_4. A committee of four decision makers or experts namely, D_1, D_2, D_3, D_4 has been formed to *select* the most appropriate location on the basis of six criteria adopted from the study conducted by Xiong et al. [90], namely, cost (C_1), distance to suppliers (C_2), distance to customers (C_3), conformance to government regulation and laws (C_4), quality of service (C_5) and environmental impact (C_6). The four decision makers use linguistic variables (see Table 1) to rating the alternatives with respect to the criterion and the decision matrices are constructed (see Table 2-5).

Table1. Conversion between linguistic variable and SVNNs

1	Very unimportant (VUI)	(.05,.25,.95)
2	Unimportant (UI)	(.25,.20,.75)
3	Medium (M)	(.50,.15,.50)
4	Important (I)	(.75,.10,.25)
5	Very important (VI)	(.95,.05,.05)

Table 2. Decision matrix for D_1 in the form of linguistic term

J_i	C_1	C_2	C_3	C_4	C_5	C_6
J_1	I	VI	I	M	M	UI
J_2	I	M	M	VI	I	I
J_3	M	VI	VI	M	I	M
J_4	VI	I	M	VI	I	I

Table 3. Decision matrix for D_2 in the form of linguistic term

J_i	C_1	C_2	C_3	C_4	C_5	C_6
J_1	I	I	I	M	I	UI
J_2	VI	I	I	I	UI	UI
J_3	UI	VI	VI	I	M	M
J_4	M	M	VI	I	VI	VI

Table 4. Decision matrix for D_3 in the form of linguistic term

J_i	C_1	C_2	C_3	C_4	C_5	C_6
J_1	I	I	VI	I	I	M
J_2	VI	M	VI	I	VI	I
J_3	UI	VI	VI	I	M	I
J_4	M	M	I	VI	VI	VI

Table 5. Decision matrix for D_4 in the form of linguistic term

J_i	C_1	C_2	C_3	C_4	C_5	C_6
J_1	VI	UI	UI	I	I	I
J_2	M	M	VI	I	M	VI
J_3	UI	VI	I	M	I	I
J_4	I	I	M	M	I	M

Step -1 Construction of the decision matrix

Decision matrix for D_1 in the form of SVNN

$$
A_1 = \begin{array}{c}
 \\ J_1 \\ J_2 \\ J_3 \\ J_4
\end{array}
\begin{pmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\
(.75, .10, .25) & (.95, .05, .05) & (.75, .10, .25) & (.50, .15, .50) & (.50, .15, .50) & (.25,.20,.75) \\
(.75, .10, .25) & (.50, .15, .50) & (.50, .15, .50) & (.95,.05, .05) & (.75, .10, .25) & (.75, .10, .25) \\
(.50, .15, .50) & (.95, .05, .05) & (.95, .05, .05) & (.50, .15, .50) & (.75, .10, .25) & (.50, .15, .50) \\
(.95, .05, .05) & (.75, .10, .25) & (.50, .15, .50) & (.95, .05, .05) & (.75, .10, .25) & (.75, .10, .25)
\end{pmatrix}
$$

Decision matrix for D_2 in the form of SVNN

$$
A_2 = \begin{array}{c}
 \\ J_1 \\ J_2 \\ J_3 \\ J_4
\end{array}
\begin{pmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\
(.75,.10,.25) & (.75,.10,.25) & (.75,.10,.25) & (.50,.15,.50) & (.75,.10,.25) & (.25,.20,.75) \\
(.95,.05,.05) & (.75,.10,.25) & (.75,.10,.25) & (.75,.10,.25) & (.25,.20,.75) & (.25,.20,.75 \\
(.25,.20,.75) & (.95,.05,.25) & (.95,.05,.05) & (.75,.10,.25) & (.50,.15,.50) & (.50,.15,.50) \\
(.50,.15,.50) & (.50,.15,.50) & (.95,.05,.05) & (.75,.10,.25) & (.95,.05,.05) & (.95,.05,.05)
\end{pmatrix}
$$

<div align="center">Decision matrix for D₃ in the form of SVNN</div>

$$
A_3 = \begin{pmatrix}
 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\
J_1 & (.75,.10,.25) & (.75,.10,.25) & (.95,.05,.05) & (.75,.10,.25) & (.75,.10,.25) & (.50,.15,.50) \\
J_2 & (.95,.05,.05) & (.50,.15,.50) & (.95,.05,.05) & (.75,.15,.25) & (.95,.05,.05) & (.75,.15,.25) \\
J_3 & (.25,.20,.75) & (.95,.05,.05) & (.95,.05,.05) & (.75,.10,.25) & (.50,.10,.50) & (.75,.10,.25) \\
J_4 & (.50,.10,.50) & (.50,.10,.50) & (.75,.10,.25) & (.95,.05,.05) & (.95,.05,.05) & (.95,.05,.05)
\end{pmatrix}
$$

<div align="center">Decision matrix for D₄ in the form of SVNN</div>

$$
A_4 = \begin{pmatrix}
 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\
J_1 & (.95,.05,.05) & (.05,.25,.95) & (.25,.20,.75) & (.75,.15,.25) & (.75,.10,.25) & (.75,.10,.25) \\
J_2 & (.50,.15,.50) & (.50,.15,.50) & (.95,.05,.05) & (.75,.10,.25) & (.50,.15,.50) & (.95,.05,.05) \\
J_3 & (.25,.20,.75) & (.95,.05,.05) & (.75,.10,.25) & (.50,.15,.50) & (.75,.10,.25) & (.75,.10,.25) \\
J_4 & (.75,.10,.25) & (.75,.10,.25) & (.50,.15,.50) & (.50,.15,.50) & (.75,.10.25) & (.50,.15,.50)
\end{pmatrix}
$$

Now we use the proposed strategy for single valued neutrosophic group decision making to select appropriate location. We take $\alpha = 0.5$ for demonstrating the computing procedure.

Step – 2 Calculate hybrid score – accuracy matrix

Hybrid score-accuracy matrix for A₁

$$
\Theta^1 = \begin{pmatrix}
 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\
J_1 & 1.45 & 1.65 & 1.450 & .925 & .925 & .40 \\
J_2 & 1.45 & .925 & .925 & 1.650 & 1.45 & 1.45 \\
J_3 & .925 & 1.65 & 1.650 & .925 & 1.45 & .925 \\
J_4 & 1.65 & 1.45 & .925 & 1.65 & 1.45 & 1.45
\end{pmatrix}
$$

Hybrid score-accuracy matrix for A₂

$$
\Theta^2 = \begin{pmatrix}
 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\
J_1 & 1.45 & 1.45 & 1.45 & .925 & 1.45 & .40 \\
J_2 & 1.65 & 1.45 & 1.45 & 1.45 & .40 & .40 \\
J_3 & .40 & 1.65 & 1.65 & 1.45 & .925 & .925 \\
J_4 & .925 & .925 & 1.65 & 1.45 & 1.65 & 1.65
\end{pmatrix}
$$

Hybrid score-accuracy matrix for A_3

$$\Theta^3 = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ J_1 & 1.45 & 1.45 & 1.65 & 1.45 & 1.45 & .925 \\ J_2 & 1.65 & .925 & 1.65 & 1.45 & 1.65 & 1.45 \\ J_3 & 1.65 & 1.65 & 1.65 & 1.45 & .925 & 1.45 \\ J_4 & .925 & .925 & 1.45 & 1.65 & 1.65 & 1.65 \end{pmatrix}$$

Hybrid score-accuracy matrix for A_4

$$\Theta^4 = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ J_1 & 1.65 & 0.40 & 0.40 & 1.45 & 1.45 & 1.45 \\ J_2 & 0.925 & 0.925 & 1.65 & 1.45 & 0.925 & 1.65 \\ J_3 & 0.40 & 1.65 & 1.45 & 0.925 & 1.45 & 1.45 \\ J_4 & 1.45 & 1.45 & 0.925 & 0.925 & 1.45 & 0.925 \end{pmatrix}$$

Step – 3 Calculate the average matrix

Using equation (7), and hybrid score-accuracy matrix, average matrix Θ^* is constructed as follows:

$$\Theta^* = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ J_1 & 1.50 & 1.24 & 1.50 & 1.19 & 1.32 & 0.79 \\ J_2 & 1.42 & 1.06 & 1.42 & 1.50 & 1.11 & 1.24 \\ J_3 & 0.84 & 1.65 & 1.60 & 1.19 & 1.19 & 1.19 \\ J_4 & 1.24 & 1.19 & 1.24 & 1.42 & 1.55 & 1.42 \end{pmatrix}$$

Using the equation (8), the collective correlation co-efficient between Θ^r and Θ^* are obtained as

$\chi_1 = 3.93$, $\chi_2 = 3.88$, $\chi_3 = 4.03$, $\chi_4 = 3.82$.

Step – 4 Determine decision makers' weights

From the equation (9) we determine the weights of the four decision makers as follows:

$\mathfrak{I}_1 = 0.250$, $\mathfrak{I}_2 = 0.248$, $\mathfrak{I}_3 = 0.257$, $\mathfrak{I}_4 = 0.244$.

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         Dr. Mohamed Abdel-Basset
         Dr. Victor Chang

Step – 5 Calculate collective hybrid score – accuracy matrix

Hence the hybrid score-accuracy values of the different decision makers' choices are aggregated by equation (11) and the collective hybrid score-accuracy matrix can be formulated as follows:

$$\Theta = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ J_1 & 1.50 & 1.24 & 1.24 & 1.19 & 1.32 & .79 \\ J_2 & 1.42 & 1.05 & 1.42 & 1.5 & 1.11 & 1.24 \\ J_3 & 0.85 & 1.65 & 1.60 & 1.19 & 1.18 & 1.19 \\ J_4 & 1.23 & 1.18 & 1.24 & 1.42 & 1.55 & 1.42 \end{pmatrix}$$

Step – 6 Weight model for criteria

Assume that incompletely known weights of the criteria are given as follows:

$0.1 \le \omega_1 \le 0.2,$ $0.1 \le \omega_2 \le 0.2,$

$0.1 \le \omega_3 \le 0.25,$ $0.1 \le \omega_4 \le 0.2,$

$0.1 \le \omega_5 \le 0.2,$ $0.1 \le \omega_6 \le 0.2$

Using the linear programming model (12), we obtain the weight vector of the criteria as

Max $=.25*((1.5* \omega_1+1.24* \omega_2+1.24* \omega_3+1.19* \omega_4+1.32* \omega_5+.79* \omega_6) +(1.42* \omega_1+1.05* \omega_2+1.42* \omega_3+1.5* \omega_4+1.11* \omega_5+1.24* \omega_6)+(.85* \omega_1+1.65* \omega_2+1.6* \omega_3+1.19* \omega_4+1.18* \omega_5+1.19* \omega_6) +(1.23* \omega_1+1.18* \omega_2+1.24* \omega_3+1.42* \omega_4+1.55* \omega_5+1.42* \omega_6));$

$\omega_1>=.1; \omega_1<=.2; \omega_2>=.1; \omega_3<=.2; \omega_3>=.1; \omega_3<=.25; \omega_4>=.1; \omega_4<=.2; \omega_5>=.1; w_5<=.2; \omega_6>=.1; \omega_6<=.2;$

$\omega_1+ \omega_2+ \omega_3+ \omega_4+ \omega_5+ \omega_6=1;$

Solutions of $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6$ are:

ω_1	0.1000000	0.000000
ω_2	0.1500000	0.000000
ω_3	0.2500000	0.000000
ω_4	0.2000000	0.000000
ω_5	0.2000000	0.000000
ω_6	0.1000000	0.000000

$\omega =[0.1, 0.15, 0.25, 0.2, 0.20, 0.1]^T.$

Step – 7 Ranking of the alternatives

Using the equation (13), we calculate the overall hybrid score-accuracy values

$\eta(J_i)$ (i = 1,2,3,4):

$\eta(J_1) = 1.227$, $\eta(J_2) = 1.300$, $\eta(J_3) = 1.326$, $\eta(J_4) = 1.346$.

Based on the above values of $\eta(J_i)$ (i = 1, 2, 3, 4), the ranking order of the locations can be presented as follows:

$J_4 > J_3 > J_2 > J_1$.

Therefore, the location J_4 is the best location.

6 Conclusion

In this article we have developed a new strategy for multi attribute group decision making by combining score and accuracy function due to Zhang et al. [71] and hybrid accuracy function due to J. Ye [67] and linguistic variables. We present a conversion between linguistic variable and SVNNs. We have also presented a numerical example for logistics center location problem using the proposed strategy under single-valued neutrosophic environment. The weights of the decision makers are completely unknown and the weights of criteria are incompletely known The proposed strategy can be used solve to multi attribute group decision making problems such as pattern recognition, medical diagnosis, personnel selection, etc. We hope that the proposed MAGDM strategy can be extended to interval neutrosophic set environment.

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II

A Scientific Decision Framework for Supplier Selection under Neutrosophic Moora Environment

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Abstract

In this paper, we present a hybrid model of Neutrosophic-MOORA for supplier selection problems. Making a suitable model for supplier selection is an important issue to amelioration competitiveness and capability of the organization, factory, project etc. selecting of the best supplier selection is not decrease delays in any organizations but also maximum profit and saving of material costs. Thus, now days supplier selection is become competitive global environment for any organization to select the best alternative or taking a decision. From a large number of availability alternative suppliers with dissimilar strengths and weaknesses for different objectives or criteria, requiring important rules or steps for supplier selection. In the recent past, the researchers used various multi criteria decision-making (MCDM) methods successfully to solve the problems of supplier selection. In this research, Multi-Objective Optimization based on Ratio Analysis (MOORA) with neutrosophic is applied to solve the real supplier selection problems. We selected a real life example to present the solution of problem that how ranking the alternative based on decreasing cost for each alternative and how formulate the problem in steps by Neutrosophic- MOORA technique.

Keywords

MOORA; Neutrosophic; Supplier selection; MCDM.

1 Introduction

The purpose of this paper is to present a hybrid method between MOORA and Neutrosophic in the framework of neutrosophic for the selection of suppliers with a focus on multi-criteria and multi-group environment. These days, Companies, organizations, factories seek to provide a fast and a good service to meet the requirements of peoples or customers [1, 2].The field of multi criteria decision-making is considered for the selection of suppliers [3]. The selecting of the best supplier increasing the efficiency of any organization whether company, factory according to [4].

Hence, for selecting the best supplier selection there are much of methodologies we presented some of them such as fuzzy sets (FS), Analytic network process (ANP), Analytic hierarchy process (AHP), (TOPSIS) technique for order of preference by similarity to ideal solution, (DSS) Decision support system, (MOORA) multi-objective optimization by ratio analysis. A classification of these methodologies to two group hybrid and individual can reported in [4, 5].

We review that the most methodologies shows the supplier selection Analytic hierarchy process (AHP), Analytic network process (ANP) with neutrosophic in [6].

1.1 Supplier Selection Problem

A Supplier selection is considered one of the most very important components of production and vulgarity management for many organizations service.

The main goal of supplier selection is to identify suppliers with the highest capability for meeting an organization needs consistently and with the minimum cost. Using a set of common criteria and measures for abroad comparison of suppliers.

However, the level of detail used for examining potential suppliers may vary depending on an organization's needs. The main purpose and objective goal of selection is to identify high-potential suppliers. To choose suppliers, the organization present judge of each supplier according to the ability of meeting the organization consistently and cost effective it's needs using selection criteria and appropriate measure.

Criteria and measures are developed to be applicable to all the suppliers being considered and to reflect the firm's needs and its supply and technology strategy.

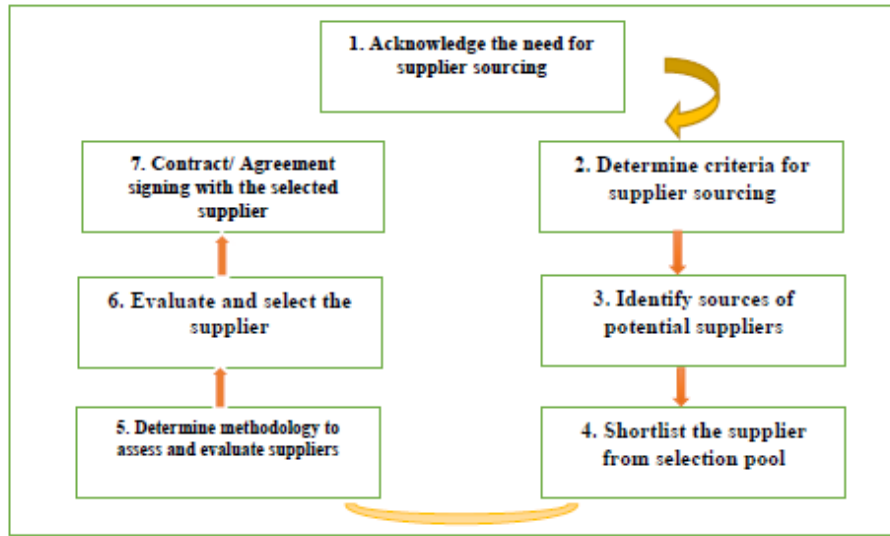We show Supplier evaluation and selection process [7].

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Figure 1. Supplier evaluation and selection process.

1.2 MOORA Technique

Multi-Objective Optimization on the basis of Ratio Analysis (MOORA), also known as multi criteria or multi attribute optimization. (MOORA) method seek to rank or select the best alternative from available option was introduced by Brauers and Zavadskas in 2006 [8].

The (MOORA) method has a large range of applications to make decisions in conflicting and difficult area of supply chain environment. MOORA can be applied in the project selection, process design selection, location selection, product selection etc. the process of defining the decision goals, collecting relevant information and selecting the best optimal alternative is known as decision making process.

The basic idea of the MOORA method is to calculate the overall performance of each alternative as the difference between the sums of its normalized performances which belongs to cost and benefit criteria.

This method applied in various fields successfully such as project management [9].

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Table 1. Comparison of MOORA with MADM approaches

MADM method	Computational Time	Simplicity	Mathematical Calculations required
MOORA	Very less	Very simple	Minimum
AHP	Very high	Very critical	Maximum
ANP	Moderate	Moderately critical	Moderate
TOPSIS	Moderate	Moderately critical	Moderate
GRA	Very high	Very critical	Maximum

1.3 Neutrosophic Theory

Smarandache first introduced neutrosophy as a branch of philosophy which studies the origin, nature, and scope of neutralities. Neutrosophic set is an important tool which generalizes the concept of the classical set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, paraconsistent set, dial theist set, paradoxist set, and tautological set[14-22]. Smarandache (1998) defined indeterminacy explicitly and stated that truth, indeterminacy, and falsity-membership are independent and lies within]-*0, 1+[.* which is the non-standard unit interval and an extension of the standard interval]-*0, 1+[.*

We present some of methodologies that it used in the multi criteria decision making and presenting the illustration between supplier selection, MOORA and Neutrosophic. Hence the goal of this paper to present the hybrid of the MOORA (Multi-Objective Optimization on the basis of Ratio Analysis) method with neutrosophic as a methodology for multi criteria decision making (MCDM).

This is ordered as follows: Section 2 gives an insight into some basic definitions on neutrosophic sets and MOORA. Section 3 explains the proposed methodology of neutrosophic MOORA model. In Section 4 a numerical example is presented in order to explain the proposed methodology. Finally, the conclusions.

2 Preliminaries

In this section, the essential definitions involving neutrosophic set, single valued neutrosophic sets, trapezoidal neutrosophic numbers and operations on trapezoidal neutrosophic numbers are defined.

2.1 Definition [10]

Let X be a space of points and $x \in X$. A neutrosophic set A in X is definite by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]$-0, 1+$[$. That is $T_A(x):X\rightarrow]$-0, 1+$[$, $I_A(x):X\rightarrow]$-0, 1+$[$ and $F_A(x):X\rightarrow]$-0, 1+$[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0- \leq \sup(x) + \sup x + \sup x \leq 3+$.

2.2 Definition [10, 11]

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object taking the form $A = \{\langle x, T_A(x), I_A(x), F_A(x), \rangle : x \in X\}$, where $T_A(x):X \rightarrow [0,1]$, $I_A(x):X \rightarrow [0,1]$ and $F_A(x):X\rightarrow[0,1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively. For convenience, a SVN number is represented by $A = (a, \text{b}, \text{c})$, where $a, b, c \in [0, 1]$ and $a+b+c \leq 3$.

2.3 Definition [12]

Suppose that $\alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \in [0,1]$ and $a_1, a_2, a_3, a_4 \in R$ where $a_1 \leq a_2 \leq a_3 \leq a_4$. Then a single valued trapezoidal neutrosophic number, $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ is a special neutrosophic set on the real line set R whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}} \left(\frac{x-a_1}{a_2-a_1}\right) & (a_1 \leq x \leq a_2) \\ \alpha_{\tilde{a}} & (a_2 \leq x \leq a_3) \\ \alpha_{\tilde{a}} \left(\frac{a_4-x}{a_4-a_3}\right) & (a_3 \leq x \leq a_4) \\ 0 & otherwise \end{cases} \quad (1).$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\theta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \leq x \leq a_2) \\ \alpha_{\tilde{a}} & (a_2 \leq x \leq a_3) \\ \frac{(x-a_3+\theta_{\tilde{a}}(a4-x))}{(a_4-a_3)} & (a_3 \leq x \leq a_4) \\ 1 & otherwise \end{cases} \quad (2).$$

$$F_{\tilde{a}}\left(x\right)=\begin{cases}\dfrac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1\leq x\leq\ a_2)\\[2mm]\alpha_{\tilde{a}} & (a_2\leq x\leq\ a_3)\\[2mm]\dfrac{(x-a_3+\beta_{\tilde{a}}(a4-x))}{(a_4-a_3)} & (a_3\leq x\leq\ a_4)\\[2mm]1 & otherwise\end{cases}\qquad(3).$$

where $\alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$ and $\beta_{\tilde{a}}$ and represent the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree respectively. A single valued trapezoidal neutrosophic number $\tilde{a}=\langle(a_1$, a_2, a_3, $a_4)$; $\alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$, $\beta_{\tilde{a}}\rangle$ may express an ill-defined quantity of the range, which is approximately equal to the interval $[a_2$, $a_3]$.

2.4 Definition [11, 10]

Let $\tilde{a}=\langle(a_1$, a_2, a_3, $a_4)$; $\alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$, $\beta_{\tilde{a}}\rangle$ and $\tilde{b}=\langle(b_1$, b_2, b_3, $b_4)$; $\alpha_{\tilde{b}}$, $\theta_{\tilde{b}}$, $\beta_{\tilde{b}}\rangle$ be two single valued trapezoidal neutrosophic numbers and $\Upsilon\neq0$ be any real number. Then,

1. Addition of two trapezoidal neutrosophic numbers

$$\tilde{a}+\tilde{b}=\langle(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle$$

2. Subtraction of two trapezoidal neutrosophic numbers

$$\tilde{a}-\tilde{b}=\langle(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle$$

3. Inverse of trapezoidal neutrosophic number

$$\tilde{a}^{-1}=\langle(\frac{1}{a_4},\frac{1}{a_3},\frac{1}{a_2},\frac{1}{a_1});\alpha_{\tilde{a}},\theta_{\tilde{a}},\beta_{\tilde{a}}\rangle\qquad\text{where }(\tilde{a}\neq0)$$

4. Multiplication of trapezoidal neutrosophic number by constant value

$$\Upsilon\tilde{a}=\begin{cases}\langle(\Upsilon a_1,\Upsilon a_2,\Upsilon a_3,\Upsilon a_4);\alpha_{\tilde{a}},\theta_{\tilde{a}},\beta_{\tilde{a}}\rangle & \text{if }(\Upsilon>0)\\\langle(\Upsilon a_4,\Upsilon a_3,\Upsilon a_2,\Upsilon a_1);\alpha_{\tilde{a}},\theta_{\tilde{a}},\beta_{\tilde{a}}\rangle & \text{if }(\Upsilon<0)\end{cases}$$

5. Division of two trapezoidal neutrosophic numbers

$$\frac{\tilde{a}}{\tilde{b}}=\begin{cases}\langle(\frac{a_1}{b_4},\frac{a_2}{b_3},\frac{a_3}{b_2},\frac{a_4}{b_1});\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle & \text{if }(a_4>0,\ b_4>0)\\\langle(\frac{a_4}{b_4},\frac{a_3}{b_3},\frac{a_2}{b_2},\frac{a_1}{b_1});\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle & \text{if }(a_4<0,\ b_4>0)\\\langle(\frac{a_4}{b_1},\frac{a_3}{b_2},\frac{a_2}{b_3},\frac{a_1}{b_4});\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle & \text{if }(a_4<0,\ b_4<0)\end{cases}$$

6. Multiplication of trapezoidal neutrosophic numbers

$$\tilde{a}\tilde{b}=\begin{cases}\langle(a_1b_1,a_2b_2,a_3b_3,a_4b_4);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle & \text{if }(a_4>0,\ b_4>0)\\\langle(a_1b_4,a_2b_3,a_3b_2,a_4b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle & \text{if }(a_4<0,\ b_4>0)\\\langle(a_4b_4,a_3b_3,a_2b_2,a_1b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle & \text{if }(a_4<0,\ b_4<0)\end{cases}$$

3 Methodology

In this paper, we present the steps of the proposed model MOORA-Neutrosophic, we define the criteria based on the opinions of decision makers (DMs) using neutrosophic trapezoidal numbers to make the judgments on criteria more accuracy, using a scale from 0 to 1 instead of the scale (1-9) that have many drawbacks illustrated by **[13]**. We present a new scale from 0 to 1 to avoid this drawbacks. We use (n-1) judgments to obtain consistent trapezoidal neutrosophic preference relations instead of $\frac{n \times (n-1)}{2}$ to decrease the workload and not tired decision makers. (MOORA-Neutrosophic) method is used for ranking and selecting the alternatives. To do this, we first present the concept of AHP to determine the weight of each criteria based on opinions of decision makers (DMs). Then each alternative is evaluated with other criteria and considering the effects of relationship among criteria.

The steps of our model can be introduced as:

Step - 1. Constructing model and problem structuring.

a. Constitute a group of decision makers (DMs).

b. Formulate the problem based on the opinions of (DMs).

Step - 2. Making the pairwise comparisons matrix and determining the weight based on opinions of (DMs).

a. Identify the criteria and sub criteria C = {C1, C2, C3…Cm}.

b. Making matrix among criteria n × m based on opinions of (DMs).

$$
W = \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ Cn \end{array}
\begin{bmatrix}
(l_{11}, m_{11l}, m_{11u}, u_{11}) & (l_{11}, m_{11l}, m_{11u}, u_{11}) & \cdots & (l_{1n}, m_{1nl}, m_{1nu}, u_{1n}) \\
(l_{21}, m_{21l}, m_{21u}, u_{21}) & (l_{22}, m_{22l}, m_{22u}, u_{22}) & \cdots & (l_{2n}, m_{2nl}, m_{2nu}, u_{2n}) \\
\cdots & \cdots & \cdots & \cdots \\
(l_{n1}, m_{n1l}, m_{n1u}, u_{n1}) & (l_{n2}, m_{n2l}, m_{n2u}, u_{n2}) & \cdots & (l_{nn}, m_{nnl}, m_{nnu}, u_{nn})
\end{bmatrix}
\quad (4)
$$

with column headers $C_1 \quad C_2 \quad \cdots \quad C_m$.

Decision makers (DMs) make pairwise comparisons matrix between criteria compared to each criterion focuses only on (n-1) consensus judgments instead of using $\frac{n \times (n-1)}{2}$ that make more workload and Difficult.

c. According to, the opinion of (DMs) should be among from 0 to 1 not negative. Then, we transform neutrosophic matrix to pairwise comparisons deterministic matrix by adding (α, θ, β) and using the following equation to calculate the accuracy and score.

$$S\left(\tilde{a}_{ij}\right) = \frac{1}{16}\left[a_1 + b_1 + c_1 + d_1\right] \times \left(2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}}\right) \tag{5}$$

and

$$A\left(\tilde{a}_{ij}\right) = \frac{1}{16}\left[a_1 + b_1 + c_1 + d_1\right] \times \left(2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} + \beta_{\tilde{a}}\right) \tag{6}$$

d.   We obtain the deterministic matrix by using $S\left(\tilde{a}_{ij}\right)$.

e.   From the deterministic matrix we obtain the weighting matrix by dividing each entry on the sum of the column.

Step - 3. Determine the decision-making matrix (DMM). The method begin with define the available alternatives and criteria

$$R = \begin{matrix} & \begin{matrix} C_1 & \qquad\qquad C_2 & \quad \cdots & \qquad C_m \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ An \end{matrix} & \begin{bmatrix} (l_{11}, m_{11l}, m_{11u}, u_{11}) & (l_{11}, m_{11l}, m_{11u}, u_{11}) & \cdots & (l_{1n}, m_{1nl}, m_{1nu}, u_{1n}) \\ (l_{21}, m_{21l}, m_{21u}, u_{21}) & (l_{22}, m_{22l}, m_{22u}, u_{22}) & \cdots & (l_{2n}, m_{2nl}, m_{2nu}, u_{2n}) \\ \cdots & \cdots & \cdots & \cdots \\ (l_{n1}, m_{n1l}, m_{n1u}, u_{n1}) & (l_{n2}, m_{n2l}, m_{n2u}, u_{n2}) & \cdots & (l_{nn}, m_{nnl}, m_{nnu}, u_{nn}) \end{bmatrix} \end{matrix} \tag{7}$$

where A_i represents the available alternatives where $i = 1\dots n$ and the C_j represents criteria

a.   Decision makers (DMs) make pairwise comparisons matrix between criteria compared to each criterion focuses only on (n-1) consensus judgments instead of using $\frac{n \times (n-1)}{2}$ that make more workload and Difficult.

b.   According to, the opinion of (DMs) should be among from 0 to 1 not negative. Then, we transform neutrosophic matrix to pairwise comparisons deterministic matrix by using equations 5 &6 to calculate the accuracy and score.

c.   We obtain the deterministic matrix by using $S\left(\tilde{a}_{ij}\right)$.

Step - 4. Calculate the normalized decision-making matrix from previous matrix (DMM).

a.   Thereby, normalization is carried out **[14].** Where the Euclidean norm is obtained according to eq. (8) to the criterionE_j.

i. $\left|Ey_j\right| = \sqrt{\sum_1^n E_i^2}$ \hfill (8)

The normalization of each entry is undertaken according to eq. (9)

ii. $\qquad NE_{ij} = \dfrac{E_{ij}}{|E_j|}$ (9)

Step - 5. Compute the aggregated weighted neutrosophic decision matrix (AWNDM) as the following:

i. $\qquad \acute{R} = R \times W$ (10)

Step - 6. Compute the contribution of each alternative Ny_i the contribution of each alternative

i. $\qquad Ny_i = \sum_{i=1}^{g} Ny_i \ - \sum_{j=g+1}^{m} Nx_j$ (11)
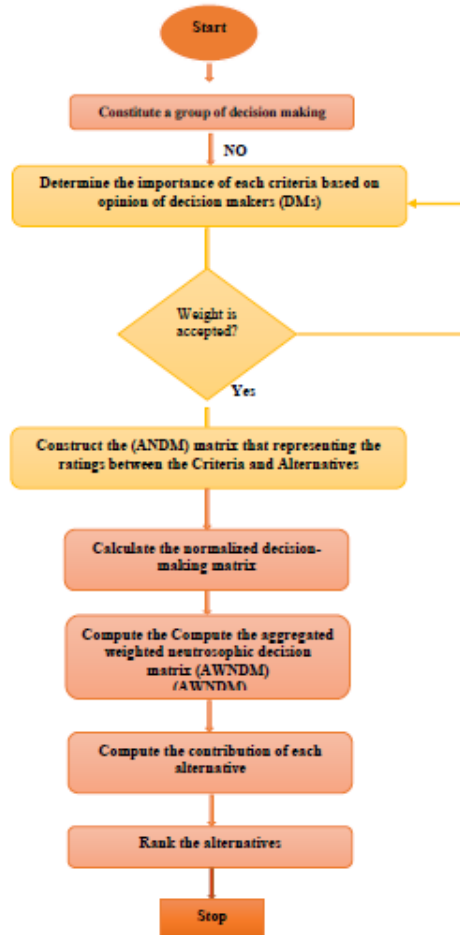
Step - 7. Rank the alternatives.

Figure 2 Schematic diagram of MOORA with neutrosophic.

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4 Implementation of Neutrosophic – MOORA Technique

In this section, to illustrate the concept of MOORA with Neutrosophic we present an example. An accumulation company dedicated to the production of the computers machines has to aggregate several components in its production line. When failure occurred from suppliers (alternatives), a company ordered from another alternative based on the four criteria C_j (j = 1, 2, 3, and 4), the four criteria are as follows: C_1 for Total Cost, C_2 for Quality, C_3 for Service, C_4 for On-time delivery. The criteria to be considered is the supplier selections are determined by the DMs from a decision group. The team is broken into four groups, namely DM_1, DM_2, DM_3 and DM_4, formed to select the most suitable alternatives. This example is that the selecting the best alternative from five alternative. A_i (i = 1, 2, 3, 4 and 5). Representing of criteria evaluation:

- Cost (C_1) Minimum values are desired.
- Quality (C_2) Maximum evaluations.
- Service (C_3) maximum evaluation.
- On-time delivery (C_4) maximum evaluation.

Step - 1. Constitute a group of decision makers (DMs) that consist of four (DM).

Step - 2. We determine the importance of each criteria based on opinion of decision makers (DMs).

$$W = \begin{array}{c c c c c} & C_1 & C_2 & C_3 & C_4 \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C4 \end{array} & \left[\begin{array}{cccc} (0.5\,,0.5,0.5,0.5) & (0.6,0.7,0.9,0.1) & (0.7\,,0.2,0.4,0.6) & (0.3\,,0.6,0.4,0.7) \\ (0.6\,,0.3,0.4,0.7) & (0.5\,,0.5,0.5,0.5) & (0.6\,,0.7,0.8,0.9) & (0.3\,,0.5,0.2,0.5) \\ (0.3\,,0.5,0.2,0.5) & (0.3\,,0.7,0.4,0.3) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.5,0.6,0.8) \\ (0.4\,,0.3,0.1,0.6) & (0.1\,,0.4,0.2,0.8) & (0.5\,,0.3,0.2,0.4) & (0.5\,,0.5,0.5,0.5) \end{array}\right] \end{array}$$

Then the last matrix appears consistent according to definition 6. And then by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), minimum indeterminacy-membership degree (θ) and minimum falsity-membership degree (β) of single valued neutrosophic numbers.

$$W = \begin{array}{c c c c c} & C_1 & C_2 & C_3 & C_4 \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C4 \end{array} & \left[\begin{array}{cccc} (0.5\,,0.5,0.5,0.5) & (0.6,0.7,0.9,0.1;0.4,0.3,0.5) & (0.7,0.2,0.4,0.6;0.8,0.4,0.2) & (0.3,0.6,0.4,0.7;0.4,0.5,0.6) \\ (0.6,0.3,0.4,0.7;0.2,0.5,0.8) & (0.5\,,0.5,0.5,0.5) & (0.6,0.7,0.8,0.9;0.2,0.5,0.7) & (0.3,0.5,0.2,0.5;0.5,0.7,0.8) \\ (0.3,0.5,0.2,0.5;0.4,0.5,0.7) & (0.3,0.7,0.4,0.3;0.2,0.5,0.9) & (0.5\,,0.5,0.5,0.5) & (0.2,0.5,0.6,0.8;0.4,0.3,0.8) \\ (0.4,0.3,0.1,0.6;0.2,0.3,0.5) & (0.1,0.4,0.2,0.8;0.7,0.3,0.6) & (0.5,0.3,0.2,0.4;0.3,0.4,0.7) & (0.5\,,0.5,0.5,0.5) \end{array}\right] \end{array}$$

From previous matrix we can determine the weight of each criteria by using the following equation of S $(\tilde{a}_{ij})$

$$S\,(\tilde{a}_{ij}) = \frac{1}{16}\,[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}})$$

and

$$A\,(\tilde{a}_{ij}) = \frac{1}{16}\,[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} + \beta_{\tilde{a}})$$

The deterministic matrix can obtain by S $(\tilde{a}_{ij})$ equation in the following step:

$$W = \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C4 \end{array}\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ \begin{bmatrix} 0.5 & 0.23 & 0.261 & 0.163 \\ 0.113 & 0.5 & 0.188 & 0.10 \\ 0.113 & 0.085 & 0.5 & 0.17 \\ 0.123 & 0.169 & 0.105 & 0.5 \end{bmatrix} \end{array}$$

From this matrix we can obtain the weight criteria by dividing each entry by the sum of each column.

$$W = \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C4 \end{array}\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ \begin{bmatrix} 0.588 & 0.234 & 0.237 & 0.175 \\ 0.133 & 0.508 & 0.171 & 0.107 \\ 0.133 & 0.086 & 0.455 & 0.182 \\ 0.145 & 0.172 & 0.095 & 0.536 \end{bmatrix} \end{array}$$

Step - 3. Construct the (ANDM) matrix that representing the ratings given by every DM between the Criteria and Alternatives.

$$\tilde{R} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}\begin{bmatrix} (0.5,0.3,0.2,0.4) & (0.6,0.7,0.9,0.1) & (0.7,0.9,1.0,1.0) & (0.4,0.7,1.0,1.0) \\ (0.0,0.1,0.3,0.4) & (0.7,0.6,0.8,0.3) & (0.6,0.7,0.8,0.9) & (0.3,0.5,0.9,1.0) \\ (0.4,0.2,0.1,0.3) & (0.3,0.0,0.5,0.8) & (0.4,0.2,0.1,0.3) & (0.2,0.5,0.6,0.8) \\ (0.7,0.3,0.3,0.6) & (0.6,0.1,0.7,1.0) & (0.2,0.4,0.5,0.8) & (0.3,0.4,0.2,0.5) \\ (0.5,0.4,0.2,0.6) & (0.4,0.6,0.1,0.2) & (0.6,0.1,0.3,0.5) & (0.7,0.1,0.3,0.2) \end{bmatrix}$$

Then the last matrix appears consistent according to definition 6. And then by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), minimum indeterminacy-membership degree (θ) and minimum falsity-membership degree (β) of single valued neutrosophic numbers.

$$C_1 \qquad\qquad C_2 \qquad\qquad C_3 \qquad\qquad C_4$$

$$R = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}\begin{bmatrix} (0.5,0.3,0.2,0.4;0.3,0.4,0.6) & (0.6,0.7,0.9,0.1;0.3,0.4,0.5) & (0.7,0.9,1.0,1.0;0.2,0.5,0.3) & (0.4,0.7,1.0,1.0;0.1,0.3,0.4) \\ (0.0,0.1,0.3,0.4;0.6,0.1,0.4) & (0.7,0.6,0.8,0.3;0.4,0.8,0.1) & (0.6,0.7,0.8,0.9;0.2,0.3,0.5) & (0.3,0.5,0.9,1.0;0.2,0.4,0.6) \\ (0.4,0.2,0.1,0.3;0.3,0.5,0.2) & (0.3,0.0,0.5,0.8;0.5,0.7,0.2) & (0.4,0.2,0.1,0.3;0.5,0.7,0.5) & (0.2,0.5,0.6,0.8;0.1,0.2,0.5) \\ (0.7,0.3,0.3,0.6;0.5,0.3,0.1) & (0.6,0.1,0.7,1.0;0.2,0.6,0.3) & (0.2,0.4,0.5,0.8;0.1,0.4,0.8) & (0.3,0.4,0.2,0.5;0.3,0.8,0.7) \\ (0.5,0.4,0.2,0.6;0.9,0.4,0.6) & (0.4,0.6,0.1,0.2;0.1,0.5,0.4) & (0.6,0.1,0.3,0.5;0.8,0.6,0.2) & (0.7,0.1,0.3,0.2;0.3,0.9,0.6) \end{bmatrix}$$

From previous matrix we can determine the weight of each criteria by using the following equation of S ($\tilde{a}_{ij}$)

$$S(\tilde{a}_{ij}) = \frac{1}{16}[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}})$$

and

$$A(\tilde{a}_{ij}) = \frac{1}{16}[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} + \beta_{\tilde{a}})$$

The deterministic matrix can obtain by S ($\tilde{a}_{ij}$) equation in the following step:

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ \begin{bmatrix} 0.11 & 0.20 & 0.32 & 0.27 \\ 0.11 & 0.23 & 0.26 & 0.20 \\ 0.10 & 0.16 & 0.08 & 0.18 \\ 0.25 & 0.19 & 0.11 & 0.07 \\ 0.20 & 0.09 & 0.19 & 0.07 \end{bmatrix} \end{array}$$

Step - 4. Calculate the normalized decision-making matrix from previous matrix.

By this equation = $|X_j| = \sqrt{\sum_1^n x_i^2}$,

$$NX_{ij} = \frac{x_{ij}}{|X_j|}$$

a. Sum of squares and their square roots

	C_1	C_2	C_3	C_4
A_1	0.11	0.20	0.32	0.27
A_2	0.11	0.23	0.26	0.20
A_3	0.10	0.16	0.08	0.18
A_4	0.25	0.19	0.11	0.07
A_5	0.20	0.09	0.19	0.07
Sum of square	0.14	0.16	0.22	0.16
Square root	0.37	0.40	0.47	0.40

b. Objectives divided by their square roots and MOORA

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ \begin{bmatrix} 0.30 & 0.50 & 0.68 & 0.67 \\ 0.30 & 0.58 & 0.55 & 0.50 \\ 0.27 & 0.40 & 0.17 & 0.45 \\ 0.68 & 0.48 & 0.23 & 0.18 \\ 0.54 & 0.23 & 0.40 & 0.18 \end{bmatrix} \end{array}$$

Step - 5. Compute the aggregated weighted neutrosophic decision matrix (AWNDM) as the following:

$$\acute{R} = \qquad R \qquad \times \qquad W$$

$$= \begin{bmatrix} 0.30 & 0.50 & 0.68 & 0.67 \\ 0.30 & 0.58 & 0.55 & 0.50 \\ 0.27 & 0.40 & 0.17 & 0.45 \\ 0.68 & 0.48 & 0.23 & 0.18 \\ 0.54 & 0.23 & 0.40 & 0.18 \end{bmatrix} \times \begin{bmatrix} 0.588 & 0.234 & 0.237 & 0.175 \\ 0.133 & 0.508 & 0.171 & 0.107 \\ 0.133 & 0.086 & 0.455 & 0.182 \\ 0.145 & 0.172 & 0.095 & 0.536 \end{bmatrix} =$$

$$= \begin{bmatrix} 0.43 & 0.20 & 0.49 & 0.59 \\ 0.40 & 0.49 & 0.47 & 0.48 \\ 0.29 & 0.59 & 0.25 & 0.36 \\ 0.52 & 0.45 & 0.36 & 0.31 \\ 0.42 & 0.31 & 0.37 & 0.29 \end{bmatrix}$$

Step - 6. Compute the contribution of each alternative Ny_i the contribution of each alternative

$$Ny_i = \sum_{i=1}^{g} Ny_i \ - \ \sum_{j=g+1}^{m} Nx_j$$

	C_1	C_2	C_3	C_4	Y_i	Rank
A_1	0.43	0.20	0.49	0.59	0,85	3
A_2	0.40	0.49	0.47	0.48	0.99	1
A_3	0.29	0.59	0.25	0.36	0.91	2
A_4	0.52	0.45	0.36	0.31	0.60	4
A_5	0.42	0.31	0.37	0.29	0.55	5

Step - 7. Rank the alternatives. The alternatives are ranked according the min cost for alternative as alternative $A_2 > A_3 > A_1 > A_4 > A_5$
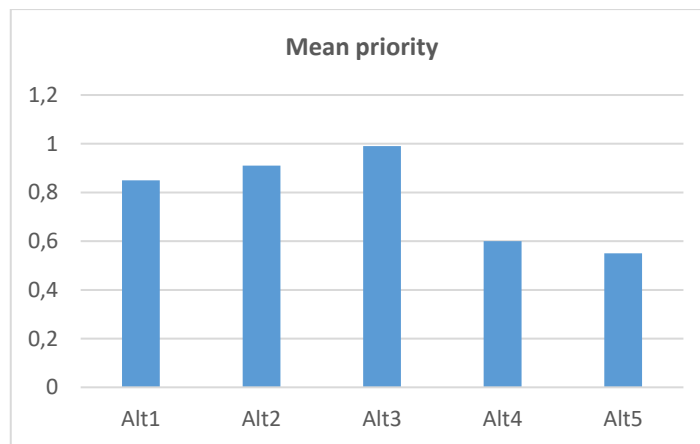

Figure 3. The MOORA- Neutrosophic ranking of alternatives.

5 Conclusion

This research presents a hybrid of the (MOORA) method with Neutrosophic for supplier selection. We presented the steps of the method in seven steps and a numerical case was presented to illustrate it. The proposed methodology provides a good hybrid technique that can facilitate the selecting of the best alternative by decision makers. Then neutrosophic provide better flexibility and the capability of handling subjective information to solve problems in the decision making. As future work, it would be interesting to apply MOORA-Neutrosophic technique in different areas as that is considered one of the decision making for selection of the best alternatives. For example, project selection, production selection, etc. The case study we presented is an example about selecting the alternative that the decision makers (DMs) specify the criteria and how select the best alternatives.

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III

Foundation of Neutrosophic Crisp Probability Theory

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Abstract

This paper deals with the application of Neutrosophic Crisp sets (which is a generalization of Crisp sets) on the classical probability, from the construction of the Neutrosophic sample space to the Neutrosophic crisp events reaching the definition of Neutrosophic classical probability for these events. Then we offer some of the properties of this probability, in addition to some important theories related to it. We also come into the definition of conditional probability and Bayes theory according to the Neutrosophic Crisp sets, and eventually offer some important illustrative examples. This is the link between the concept of Neutrosophic for classical events and the neutrosophic concept of fuzzy events. These concepts can be applied in computer translators and decision-making theory.

Keywords

Neutrosophic logic; fuzzy logic; classical logic; classical probability; Neutrosophic Crisp sets.

1 Introduction

The Neutrosophic logic is non-classical and new logic founded by the philosopher and mathematical American Florentin Smarandache in 1999. In [6] Salama introduced the concept of neutrosophic crisp set Theory, to represent any event by a triple crisp structure. Moreover the work of Salama et al. [1-10] formed a starting point to construct new branches of neutrosophic mathematics and computer sci. Hence, Neutrosophic set theory turned out to be a generalization of both the classical and fuzzy counterparts. When he presented it

as a generalization of the Fuzzy logic, and an extension of the Fuzzy Sets Theory [9] presented by Zadeh in 1965 Played an important role in expanding our scientific and practical approach and reducing the degree of randomization in data that helps us reach high-resolution results. An extension of that logic was introduced by A.A. Salama, the Neutrosophic crisp set theory as a generalization of classical set theory and Neutrosophic logic is a new branch that studies the origin, nature, and field of indeterminacy, as well as the interaction of all the different spectra imaginable in a case. This logic takes into account each idea with its antithesis with the indeterminacy spectrum. The main idea of Neutrosophic logic is to distinguish every logical statement in three dimensions[3.10] are truth in degrees (T) , false in degrees (F) and indeterminacy in degrees (I) we express it in form (T, I, F) and puts them under the field of study, which gives a more accurate description of the data of the phenomenon studied, as this reduces the degree of randomization in the data, which will reach high-resolution results contribute to the adoption of the most appropriate decisions among decision makers. The Neutrosophyis a word composed of two sections :Neutro (in French Neutre, in LatinNeuter) meaning Neutral, and SophyIt is a Greek word meaning wisdom and then the meaning ofthe word in its entirety (knowledge of neutral thought). We note that classical logic studies the situation with its opposite without acknowledging the state of indeterminacy, which is an explicit quantity in the logic of Neutrosophic and one of its components, which gives a more accurate description of the study and thus obtain more correct results. -In this paper we present a study of the application of the Neutrosophic logic to the classical possibilities ‹from the occurrence of the experiment to the creation of probability and then to study its properties.

2 Terminologies

2.1 Neutrosophic Random Experiments

We know the importance of experiments in the fields of science and engineering. Experimentation is useful in use, assuming that experiments under close conditions will yield equal results.

In these circumstances, we will be able to determine the values of variables that affect the results of the experiment. In any case, in some experiments, we cannot determine the values of some variables and therefore the results will change from experiment to other.

However, most of the conditions remain as it is. These experiments are described as randomized trials. When we get an undetermined result in the experiment (indeterminacy) and we take and acknowledge this result, we have a neutrosophic experience.

2.2 Example

When throwing the dice, the result we will get from the experiment is one of the following results: {1, 2, 3, 4, 5, 6, i} Where i represents an indeterminacy result. We call this experience a Neutrosophic randomized experiment.

2.3 Sample Spaces and Events due to Neutrosophic

Group X consists of all possible results of a randomized experiment called the sample space. When these results include the result of the indeterminacy, we obtain the Neutrosophic sample space.

2.4 Neutrosophic events

The event: Is a subset A of the sample space X, that is, a set of possible outcomes. The Neutrosophic set of the sample space formed by all the different assemblies (which may or may not include indeterminacy) of the possible results these assemblies are called Neutrosophic. Salama and Hanafy et al. [12-14] introduced laws to calculate correlation coefficients and study regression lines for the new type of data; a new concept of probability has been introduced for this kind of events. It is a generalization of the old events and the theory of the ancient possibilities. This is the link between the concept of Neutrosophic for classical events and the neutrosophic concept of fuzzy events. These concepts can be applied in computer translators and decision-making theory.

2.5 The concept of Neutrosophic probability

We know that probability is a measure of the possibility of a particular event, and Smarandache presented the neutrosophic experimental probability, which is a generalization of the classical experimental probability as follows [2, 4]:

$$\left(\frac{number\ of\ times\ event\ A\ occurs}{total\ number\ of\ trials}, \frac{number\ of\ times\ indeterminacy\ occur}{total\ number\ of\ trials}, \frac{number\ of\ times\ event\ A\ does\ not\ occur}{total\ number\ of\ trials} \right)$$

If we had the neutrosophic event, $A = (A_1, A_2, A_3)$ we define the neutrosophic probability (Which is marked with the symbol NP) for this event as follows:

$NP(A) = \left(P(A_1), P(A_2), P(A_3) \right) = (T, I, F)$, with:

$P(A_1)$ represents the probability of event A

$P(A_2)$ represents the probability of *indeterminacy*

$P(A_3)$ represents the probability that event A will not occur

According to the definition of classical probability: $P_1, P_2, P_3 \in [0,1]$
We therefore define the neutrosophic probability [2] in the form:

$NP: X \rightarrow [0,1]^3$, where X is a neutrosophic sample space.

The micro-space of the total group, which has a neutrosophic probability for each of its partial groups, calls it a neutrosophic classical probability space.

In [7, 13] the neutrosophic logic can distinguish between the absolutely sure event (the sure event in all possible worlds and its probabilistic value is 1+) and the relative sure event (the sure event in at least one world and not in all worlds its probability is 1) where $1 < 1^+$. Similarly, we distinguish between the absolutely impossible event (the impossible event in all possible worlds its probabilistic value is -0) and the relative impossible event (the impossible event in at least one world and not in all worlds its probabilistic value is 0) where -0 <0.

$^-0 = 0 - \varepsilon \,\&\, 1^+ = 1 + \varepsilon$ where ε is a very small positive number.

So, define components (T, I, F) on the non-standard domain]-0 , 1+ [.

For $A = (A_1, A_2, A_3)$ neutrosophic classical event Then it is:
$$^-0 \leq P(A_1) + P(A_2) + P(A_3) \leq 3^+$$

For $A = (A_1, A_2, A_3)$ neutrosophic crisp event of the first type Then :
$$0 \leq P(A_1) + P(A_2) + P(A_3) \leq 2$$

The probability of neutrosophic crisp event of the second type is a neutrosophic crisp event then :
$$^-0 \leq P(A_1) + P(A_2) + P(A_3) \leq 2^+$$

The probability of neutrosophic crisp event of the third type is a neutrosophic crisp event then:
$$^-0 \leq P(A_1) + P(A_2) + P(A_3) \leq 3^+ \qquad \text{...........} [12]$$

2.6 The Axioms of Neutrosophic probability

For $A = (A_1, A_2, A_3)$ neutrosophic crisp event on the X then :

$$NP(A) = \big(P(A_1), P(A_2), P(A_3) \big)$$

where:
$$P(A_1) \geq 0 \quad, \quad P(A_2) \geq 0 \quad, \quad P(A_3) \geq 0$$

The probability of neutrosophic crisp event $A = (A_1, A_2, A_3)$
$$NP(A) = \big(P(A_1), P(A_2), P(A_3) \big)$$

Where:
$$0 \leq P(A_1) \leq 1 \quad, \quad 0 \leq P(A_2) \leq 1 \quad, \quad 0 \leq P(A_3) \leq 1$$

For $A_1, A_2, \dots\dots$ Inconsistent neutrosophic crisp events then :

$$NP(A) = (A_1 \cup A_2 \cup \dots\dots) = \big(\; P(A_1) + P(A_2) +$$

$$\dots\dots \; , \; P(i_{A_1 \cup A_2 \cup \dots}) \, , \, p(\overline{A_1 \cup A_2 \cup \dots}) \big).$$

3 Some important theorems on the neutrosophic crisp probability

Theorem 1

If we have A, B two neutrosophic crisp events and $A \subseteq B$ then:

The first type:

$$NP(A) \leq NP(B) \quad \Leftrightarrow \quad P(A_1) \leq P(B_1), \quad P(A_2) \leq P(B_2),$$
$$P(A_3) \geq P(B_3)$$

The second type:

$$NP(A) \leq NP(B) \quad \Leftrightarrow \quad P(A_1) \leq P(B_1), \quad P(A_2) \geq P(B_2),$$
$$P(A_3) \geq P(B_3)$$

Theorem 2

Probability of the neutrosophic impossible event (symbolized by form $NP(\emptyset_N)$) we define it as four types:

The first type:

$$NP(\emptyset_N) = \big(P(\emptyset), P(\emptyset), P(\emptyset)\big) = (0,0,0) = 0_N$$

The second type:

$$NP(\emptyset_N) = \big(P(\emptyset), P(\emptyset), P(X)\big) = (0,0,1)$$

The third type:

$$NP(\emptyset_N) = \big(P(\emptyset), P(X), P(\emptyset)\big) = (0,1,0)$$

The fourth type:

$$NP(\emptyset_N) = \big(P(\emptyset), P(X), P(X)\big) = (0,1,1)$$

Theorem 3

Probability of the neutrosophic overall crisp event (symbolized by form $NP(X_N)$) we define it as four types:

The first type:

$$NP(X_N) = \big(P(X), P(X), P(X)\big) = (1,1,1) = 1_N$$

The second type:

$$NP(X_N) = \big(P(X), P(X), P(\emptyset)\big) = (1,1,0)$$

The third type:

$$NP(X_N) = \big(P(X), P(\emptyset), P(\emptyset)\big) = (1,0,0)$$

The fourth type:

$$NP(X_N) = \big(P(X), P(\emptyset), P(X)\big) = (1,0,1)$$

Theorem 4

If A^c represents the complement of the event A, then the probability of this event is given according to the following may be three types:

Where $A^c = (A_1^c, A_2^c, A_3^c)$

The first type:

$$NP(A^c) = \big(P(A_1^c), P(A_2^c), P(A_3^c)\big)$$
$$= (1 - p(A_1), 1 - p(A_2), 1 - p(A_3))$$

The second type:

$$NP\ (A^c) = (\ P(A_3)\ , P(A_2)\ , P(A_1)\ )$$

The third type:

$$NP\ (A^c) = (\ P(A_3)\ , P(A_2^c)\ , P(A_1)\ )$$

Theorem 5

For A, B two neutrosophic crisp events

$$A = (A_1, A_2, A_3)$$
$$B = (B_1, B_2, B_3)$$

Then the probability of the intersection of these two events is given in the form:

$$NP(A \cap B) = (\ P(A_1 \cap B_1)\ , P(A_2 \cap B_2), P(A_3 \cup B_3)\ )$$

or

$$NP(A \cap B) = (\ P(A_1 \cap B_1)\ , P(A_2 \cup B_2), P(A_3 \cup B_3)\ )$$

In general if we have the neutrosophic crisp events A, B, C then:

$$NP(A \cap B \cap C) = (\ P(A_1 \cap B_1 \cap C_1)\ , P(A_2 \cap B_2 \cap C_2), P(A_3 \cup B_3 \cup C_3)\ )$$

Or

$$NP(A \cap B \cap C) = (\ P(A_1 \cap B_1 \cap C_1)\ , P(A_2 \cup B_2 \cup C_2), P(A_3 \cup B_3 \cup C_3)\ )$$

We can generalize on n of the neutrosophic crisp events.

Theorem 6

Under the same assumptions in theory (1-5) the union of these two neutrosophic crisp events will be: [28]

$$NP(A \cup B) = (\ P(A_1 \cup B_1)\ , P(A_2 \cup B_2), P(A_3 \cap B_3)\ )\ \text{Or}$$
$$NP(A \cup B) = (\ P(A_1 \cup B_1)\ , P(A_2 \cap B_2), P(A_3 \cap B_3)\ )$$

Theorem 7

If we have a neutrosophic crisp event that is about:

$$A = A_1 \cup A_2 \cup \dots \dots \dots \cup A_n$$

The neutrosophic crisp events $A_1, A_2, \dots \dots, A_n$ are In consistent then neutrosophic crisp event A we write it in the form:

$$A = (A_1, A_2, A_3)$$
$$= ((A_{11}, A_{12}, A_{13}) \cup (A_{21}, A_{22}, A_{23}) \cup \dots \dots$$
$$\cup (A_{n1}, A_{n2}, A_{n3}))$$

Therefore:

$$NP(A) = NP(A_1) + NP(A_2) + \dots \dots + NP(A_n)$$

Theorem 8

If we have A neutrosophic crisp event and A^c It is an complement event on the whole set X then:

$A \cup A^c = X$ Therefore:

$$NP\ (A) + NP(A^c) = NP(X_N) = 1_N = (1,1,1)$$

4 Neutrosophic Crisp Conditional Probability

If we have A, B two neutrosophic crisp events
$$B = (B_1, B_2, B_3) \qquad A = (A_1, A_2, A_3)$$
Then the neutrosophic conditional probability is defined to occur A if B occurs in the form:
$$NP(A|B) = \left(P(A|B), P(indeter_{A|B}), P(A^c|B) \right)$$
IF: $P(B) > 0$ $\quad = (\frac{p(A \cap B)}{P(B)}, P(indeter_{A|B}), \frac{p(A^c \cap B)}{P(B)})$
From it we conclude that:
$$NP(A|B) \neq NP(B|A)$$

- The conditional probability of complement the neutrosophic event A^c is conditioned by the occurrence of the event B.

We distinguish it from the following types*:*

The first type:
$$NP(A^c|B) = (\frac{P(A_3 \cap B_1)}{P(B_1)}, \frac{P(A_2^c \cap B_2)}{P(B_2)}, \frac{P(A_1 \cap B_3)}{P(B_3)})$$

The second type:
$$NP(A^c|B) = (\frac{P(A_3 \cap B_1)}{P(B_1)}, \frac{P(A_2 \cap B_2)}{P(B_2)}, \frac{P(A_1 \cap B_3)}{P(B_3)})$$

- The rule of multiplication in neutrosophic crisp conditional probability:
$$NP(A \cap B)$$
$$= (P(A_1).P(B_1|A_1), P(A_2).P(B_2|A_2), P(A_3).P(B_3^c|A_3))$$

5 Independent Neutrosophic Events

We say of the neutrosophic events that they are independent if the occurrence of either does not affect the occurrence of the other. Then the neutrosophic conditional probability of the crisp event A condition of occurrence B is it neutrosophic crisp probability of A. We can verify independence of A, B if one of the following conditions is check:
$$NP(A|B) = NP(A), NP(B|A) = NP(B), NP(A \cap B) = NP(A).NP(B)$$
(We can easily validate the above conditions based on classical conditional probability) .

Equally:

If the two neutrosophic crisp events A, B are independent then:

A^c Independent of B

A Independent of B^c

A^c Independent of B^c

(Pronounced from the definition of a complementary event in Theorems 4).

6 The law of total probability and Bayes theorem via Neutrosophic crisp sets

6.1 The law of Neutrosophic crisp total probability

(1) We have a sample space consisting of then neutrosophic crisp comprehensive events $A_1, A_2 \ldots \ldots, A_n$
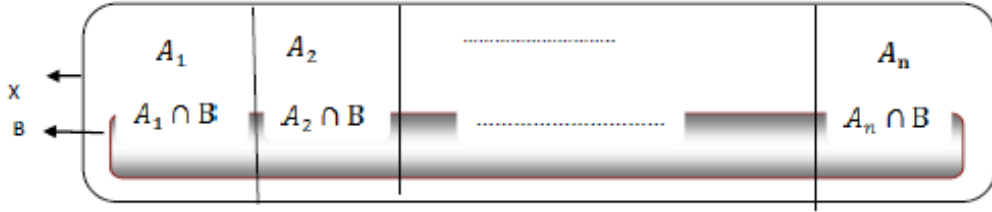
$$A_1 \cup A_2 \cup \ldots \cup A_n = X_N$$

$$\left((A_{11}, A_{12}, A_{13}) \cup (A_{21}, A_{22}, A_{23}) \cup \ldots \ldots \cup (A_{n1}, A_{n2}, A_{n3}) \right) = X_N$$

(2) The neutrosophic comprehensive events are inconsistent two at a time among them:

$$A_i \cap A_j = \emptyset \quad \forall i \neq j$$

(3) The neutrosophic crisp event B represents a common feature in all joint neutrosophic crisp events , note the following figure(1):

Figure (1)

We take the neutrosophic crisp probability for these events:

$$NP(A_1), NP(A_2), \ldots \ldots, NP(A_n)$$

From the graphic, we note that:

$$NP(B) = NP(A_1 \cap B) + NP(A_2 \cap B) + \cdots \ldots + NP(A_n \cap B)$$

From the definition of neutrosophic crisp conditional probability:

$$NP(B \cap A_i) = \left(P(A_{i\,1}).P(B \backslash A_{i\,1}), P(A_{i\,2}).P(B \backslash A_{i\,2}), P(A_{i\,3}).P(B \backslash A_{i\,3}) \right)$$

Therefore:

$$NP(B) = NP(B|A_1).NP(A_1) + NP(B|A_2).NP(A_2)$$
$$+ \ldots \ldots + NP(B|A_n).NP(A_n)$$

Which is equal to

$$= (p(A_{11}).P(B \backslash A_{11}), p(A_{12}).P(B \backslash A_{12}), p(A_{13}).P(B^c \backslash A_{13}) +$$
$$p(A_{21}).P(B \backslash A_{21}), p(A_{22}).P(B \backslash A_{22}), p(A_{23}).P(B^c \backslash A_{23}) +$$
$$+ \cdots \ldots \ldots + (p(A_{n1}).P(B \backslash A_{n1}), p(A_{n2}).P(B \backslash A_{n2}), p(A_{n3}).P(B^c \backslash A_{n3})$$

6.2 Bayes theorem by Neutrosophic:

Taking advantage of the previous figure (1):

Neutrosophic total probability iff Probability of occurrence a common feature B.

Bayes theorem iff provided that the neutrosophic crisp event occur B,What is the probability of being from A_i (Item selected from B, What is the probability of being from A_i)

Under the same assumptions that we have set in the definition of the law of neutrosophic crisp total probability, we reach the Bayes Law as follows:

$$NP(A_i \backslash B) = (\frac{P(B_1 \backslash A_{i1})p(A_{i1})}{p(B_1)}, \frac{P(B_2 \backslash A_{i2})p(A_{i2})}{p(B_2)}, \frac{P(B_3 \backslash A_{i3}^c)p(A_{i3}^c)}{p(B_3)})$$

6.3 Examples

Let us have the experience of throwing a dice stone and thus we have the neutrosophic sample space as: X= {1, 2, 3, 4, 5, 6, i}, where i represents the probability of getting indeterminacy.

We have the possibility of getting indeterminacy= 0.10

Then to calculate the following possibilities:

1- $NP(1) = (\frac{1-0.10}{6}, 0.10, 5.\frac{1-0.10}{6})$

$= (\ 0.15\ , 0.10\ , 0.75\ ) = \ NP\ (\ 2\ ) = \cdots \ldots = \ NP(\ 6\ )$

2- $NP\ (1^c) = (\ P(2,3,4,5)\ ,\ 0.10\ ,\ P(1))$

$= (\ 5\ (0.15)\ ,\ 0.10\ ,\ 0.15\ ) = (\ 0.75\ ,\ 0.10\ , 0.15\ )$

3- $NP(1\ or\ 2) = (\ p(1) + p(2)\ , 0.10\ , p(3,4,5,6))$

$= (\ 2(0.15)\ , 0.10\ , 4(0.15)) = (\ 0.30\ , 0.10\ , 0.60\ )$

But when we have B={2,3,4,5} , A={1,2,3 } then :

$NP(A\ or\ B) = (\ P(A) + \ P(B) - P(A \cap B)\ , 0.10\ , P(A^c)\ and\ P(B^c))$

$= (\ 3(0.15) + 4(0.15) - 2(0.15)\ ,\qquad 0.10\ , P\{4,5,6\}\ and\ P\{1, 6\})$

$= (\ 0.75, 0.10\ , P(6)) = (0.75\ , 0.10\ , 0.15\ )$

4- $NP(\{1,2,3\}) = (P\{1,2,3\}, 0.10, P\{1,2,3\}^c)$

$= (\ p(1) + p(2) + p(3)\ , 0.10\ , p(4) + p(5) + p(6))$

$= (0.15 + 0.15 + 0.15\ ,\qquad 0.10, 0.15 + 0.15 + 0.15)$

$= (\ 0.45\ , 0.10\ , 0,45\ )$

I. Assuming we have a jar containing:

5 cards have a symbol A, 3cards have a symbol B

2 cards are not specified (The symbol is erased on them)

If A represents is getting the card A from the jar

B represents is getting the card B from the jar

Then

$NP\ (A) = (\frac{5}{10}, \frac{2}{10}, \frac{3}{10})\qquad,\quad NP\ (B) = (\frac{3}{10}, \frac{2}{10}, \frac{5}{10})$

If card B is withdrawn from the jar then it will be:

$$NP\ (A\backslash B) = \left(\frac{P(B\backslash A).P(A)}{P(B)}\ ,\qquad P\big(indeter_{\ A\backslash B}\big),\qquad P(A^c\backslash B)\right)$$

$$= \left(\frac{\left(\frac{3}{9}\right)\left(\frac{5}{9}\right)}{\frac{3}{9}}\ ,\qquad \frac{2}{9}\ ,\qquad P(B\backslash B) = P(B) = \frac{2}{9}\right) = (\frac{5}{9},\frac{2}{9},\frac{2}{9})$$

If card A is withdrawn from the jar then it will be:

The same way we get: $NP(B\backslash A) = (\ \frac{3}{9}\ ,\frac{2}{9}\ ,\frac{4}{9}\ )$

Thus, Bayes theory according to neutrosophic be as:

$$NP(A\backslash B) = \left(\ P(A\backslash B\ ), P\big(indeter_{\ A\backslash B}\big), P(A^c\backslash B)\right)$$

$$= \frac{P(B\backslash A).P(A)}{P(B)}\ ,\ P\big(indeter_{\ A\backslash B}\big)\ ,\frac{P(B\backslash A^c).P(A^c)}{P(B)}\ )$$

$$= \left(\frac{3}{9}\frac{\frac{5}{10}}{\frac{3}{10}}\ ,\qquad \frac{2}{9}\ ,P(B\backslash B)\right) = (\frac{5}{9},\frac{2}{9},\frac{2}{9})$$

Let us have the X set X={ a ,b ,c ,d } and

A= ({a,b} , {c} , {d})

B= ({a}, {c}, {d,b})

Two neutrosophic events from the first type on X and we have:

$$U_1 = (\{a, b\}, \{c, d\}, \{a, d\})$$
$$U_2 = (\{a, b, c\}, \{c\}, \{d\})$$

Two neutrosophic events from the third type on X then:

The first type :

$$A \cap B = (\ \{a\}, \{c\}, \{d, b\})$$
$$NP(A \cap B) = (\ 0.25\ , 0.25\ , 0.50\ )$$

The second type:

$$A \cap B = (\ \{a\}, \{c\}, \{d, b\})$$
$$NP(A \cap B) = (\ 0.25\ , 0.25\ , 0.50\ )$$

The first type :

$$A \cup B = (\ \{a, b\}, \{c\}, \{d\})$$
$$NP(A \cup B) = (\ 0.50\ , 0.25\ , 0.25\ )$$

The second type:

$$A \cup B = (\ \{a, b\}, \{c\}, \{d\})$$
$$NP(A \cup B) = (\ 0.50\ , 0.25\ , 0.25\ )$$

The first type :

$$A^c = (\{c, d\}, \{a, b, d\}, \{a, b, c\})$$
$$NP(A^c) = (\ 0.50\ , 0.75\ , 0.75\ )$$

The second type:

$$A^c = (\{d\}, \{c\}, \{a, b\})$$

$$NP(A^c) = (\,0.25\,,0.25\,,0.50\,)$$

The third type:
$$A^c = (\{d\},\{a,b,d\},\{a,b\})$$
$$NP(A^c) = (\,0.25\,,0.75\,,0.50\,)$$

The first type :
$$B^c = (\{b,c,d\},\{a,b,d\},\{a,b\})$$
$$NP(B^c) = (\,0.75\,,0.75\,,0.50\,)$$

The second type
$$B^c = (\{b,d\},\{c\},\{a\})$$

The third type:
$$B^c = (\{b,d\},\{a,b,d\},\{a\})$$
$$NP(B^c) = (\,0.50\,,0.75\,,0.25)$$

The first type
$$U_1 \cup U_2 = (\{\,a,b,c\},\{\,c,d\},\{d\})$$
$$NP(U_1 \cup U_2) = (\,0.75\,,0.50\,,0.25\,)$$

The second type
$$U_1 \cup U_2 = (\{\,a,b,c\},\{\,c\},\{d\})$$
$$NP(U_1 \cup U_2) = (\,0.75\,,0.25\,,0.25\,)$$

The first type
$$U_1 \cap U_2 = (\{\,a,b\},\{\,c\},\{a,d\})$$
$$NP(U_1 \cap U_2) = (\,0.50\,,0.25\,,0.50\,)$$

The second type
$$U_1 \cap U_2 = (\{\,a,b\},\{\,c,d\},\{a,d\})$$
$$NP(U_1 \cap U_2) = (\,0.50\,,0.50\,,0.50\,)$$

The first type:
$$U_1^c = (\{c,d\},\{a,b\},\{b,c\})$$
$$NP\,(U_1^c) = (0.50\,,0.50\,,0.50\,)$$

The second type:
$$U_1^c = (\{a,d\},\{c,d\},\{a,b\})$$
$$NP\,(U_1^c) = (0.50\,,0.50\,,0.50\,)$$

The third type
$$U_1^c = (\{a,d\},\{a,b\},\{a,d\})$$
$$NP\,(U_1^c) = (0.50\,,0.50\,,0.50\,)$$

The first type:
$$U_2^c = (\{d\},\{a,b,d\},\{a,b,c\})$$
$$NP\,(U_2^c) = (0.25\,,0.75\,,0.75\,)$$

The second type:
$$U_2^c = (\{d\},\{c\},\{a,b,c\})$$
9- $NP\,(U_2^c) = (0.25\,,0.25\,,0.75\,)$

The third type

$$U_2^c = (\{d\}, \{a, b, d\}, \{a, b, c\})$$
$$NP\ (U_2^c) = (0.25\ , 0.75\ , 0.75\ )$$

$NP\ (A) = (0.50, 0.25, 0.25)$

$NP(B) = (0.25, 0.25, 0.50)$

$NP\ (U_1) = (0.50, 0.50, 0.50)$

$NP\ (U_2) = (0.75, 0.25, 0.25)$

$NP\ (U_1^c\ ) = (0.50, 0.50, 0.50)$

$NP\ (U_2^c\ ) = (0.25, 0.75, 0.75)$

10- $(A \cap B)^c = (\{b, c, d\}, \{a, b, d\}, \{a, c\})$

$$NP(A \cap B)^c = (0.75\ , 0.75\ , 0.50\ )$$

11- $NP\ (A^c)\ \cap NP(B^c) = (\{c, d\}, \{a, b, d\}, \{a, b, c\})$

$$= (\ 0.50\ , 0.75\ , 0.75\ )$$

$$NP\ (A^c)\ \cup NP(B^c) = (\{c, d, b\}, \{a, b, d\}, \{a, b, c\})$$

$$= (0.75\ , 0.75\ , 0.75\ )$$

12- $A * B = \{(a, a), (b, a)\}, \{(c, c)\}, \{(d, d), (d, b)\}$

$$NP(A * B) = (\frac{2}{16}, \frac{1}{16}, \frac{2}{16})$$

$$B * A = (\{(a, a), (a, b)\}, \{c, c\}, \{(d, d), (b, d)\})$$

$$NP(B * A) = (\frac{2}{16}, \frac{1}{16}, \frac{2}{16})$$

$$A * U_1 = \{(a, a), (a, b), (b, a\ ), (b, b)\}, \{(c, c), (c, d)\}, \{(d, a), (d, d\ )\}$$

$$NP(A * U_1) = (\frac{4}{16}, \frac{2}{16}, \frac{2}{16})$$

$U_1 * U_2$
$$= (\{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c)\}, \{(c, c), (d, c)\}, \{(a, d), (d, d)\}$$

$$= (\frac{6}{16}, \frac{2}{16}, \frac{2}{16})$$

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IV

A Novel Methodology Developing an Integrated ANP: A Neutrosophic Model for Supplier Selection

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Abstract

In this research, the main objectives are to study the Analytic Network Process (ANP) technique in neutrosophic environment, to develop a new method for formulating the problem of Multi-Criteria Decision-Making (MCDM) in network structure, and to present a way of checking and calculating consistency consensus degree of decision makers. We have used neutrosophic set theory in ANP to overcome the situation when the decision makers might have restricted knowledge or different opinions, and to specify deterministic valuation values to comparison judgments. We formulated each pairwise comparison judgment as a trapezoidal neutrosophic number. The decision makers specify the weight criteria in the problem and compare between each criteria the effect of each criteria against other criteria. In decision-making process, each decision maker should make $\frac{n \times (n-1)}{2}$ relations for n alternatives to obtain a consistent trapezoidal neutrosophic preference relation. In this research, decision makers use judgments to enhance the performance of ANP. We introduced a real life example: how to select personal cars according to opinions of decision makers. Through solution of a numerical example, we formulate an ANP problem in neutrosophic environment.
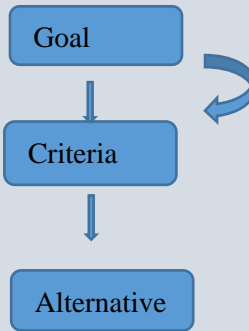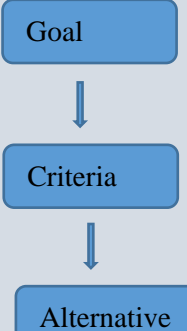
Keywords

Analytic Network Process, Neutrosophic Set, Multi-Criteria Decision Analysis (MCDM).

1 Introduction

The Analytic Network Process (ANP) is a new theory that extends the Analytic Hierarchy Process (AHP) to cases of dependency and feedback, and generalizes the supermatrix approach introduced by Saaty (1980) for the AHP [1]. This research focuses on ANP method, which is a generalization of AHP. Analytical Hierarchy Process (AHP) [2] is a multi-criteria decision making method where, given the criteria and alternative solutions of a specific model, a graph structure is created, and the decision maker is asked to pair-wisely compare the components, in order to determine their priorities. On the other hand, ANP supports feedback and interaction by having inner and outer dependencies among the models' components [2]. We deal with the problem, analyze it, and specify alternatives and the critical factors that change the decision. ANP is considered one of the most adequate technique for dealing with multi criteria decision-making using network hierarchy [19]. We present a comparison of ANP vs. AHP in *Table 1*: how each technique deals with a problem, the results of each technique, advantages and disadvantages.

Table1. Comparison of ANP vs. AHP.

Property	ANP (Analytic Network Process)	AHP (Analytic Hierarch Process)
Structure	Network	Hierarchy

	The user learns through feedback comparisons that his/her priority for cost is not nearly as high as originally thought when asked the question abstractly, while prestige gets more weight.	The user going top down makes comparisons, when asked, without referring to actual alternatives, and overestimates the importance of cost.
Why are the results different?		
Advantages	a) Using feedback and interdependence between criteria. b) Deal with complex problem without structure.	a) Straightforward and convenient. b) Simplicity by using pairwise comparisons.
Disadvantages	a) Conflict between decision makers. b) Inconsistencies. c) Hole of large scale 1 to 9. d) Large comparisons matrix.	a) Decision maker's capacity. b) Inconsistencies. c) Hole of large scale 1 to 9. d) Large comparisons matrix.

Analytic network process (ANP) consists of criteria and alternatives by decomposing them into sub-problems, specifying the weight of each criterion and comparing each criterion against other criterion, in a range between 0 and 1. We employ ANP in decision problems, and we make pairwise comparison matrices between alternatives and criteria. In any traditional methods, decision makers face a difficult problem to make $\frac{n \times (n-1)}{2}$ consistent judgments for each alternative.

In this article, we deal with this problem by making decision maker using (n-1) judgments. The analysis of ANP requires applying a scale system for pairwise comparisons matrix, and this scale plays an important role in transforming qualitative analysis to quantitative analysis [4].

Most of previous researchers use the scale 1-9 of analytic network process and hierarchy. In this research, we introduced a new scale from 0 to 1, instead of the scale 1-9. This scale 1-9 creates large hole between ranking results, and we overcome this drawback by using the scale [0, 1] [5], determined by some serious mathematical shortages of Saaty's scale, such as:

- Large hole between ranking results and human judgments;
- Conflicting between ruling matrix and human intellect.

The neutrosophic set is a generalization of the intuitionistic fuzzy set. While fuzzy sets use true and false for express relationship, neutrosophic sets use true membership, false membership and indeterminacy membership [6]. ANP employs network structure, dependence and feedback [7]. MCDM is a formal and structured decision making methodology for dealing with complex problems [8]. ANP was also integrated as a SWOT method [9]. An overview of integrated ANP with intuitionistic fuzzy can be found in Rouyendegh, [10].

Our research is organized as it follows: Section 2 gives an insight towards some basic definitions of neutrosophic sets and ANP. Section 3 explains the proposed methodology of neutrosophic ANP group decision making model. Section 4 introduces a numerical example.

2 Preliminaries

In this section, we give definitions involving neutrosophic set, single valued neutrosophic sets, trapezoidal neutrosophic numbers, and operations on trapezoidal neutrosophic numbers.

2.1 Definition [26-27]

Let X be a space of points and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]$-0, 1+[. That is $T_A(x):X \rightarrow]$-0, 1+[, $I_A(x):X \rightarrow]$-0, 1+[and $F_A(x):X \rightarrow]$-0, 1+[. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0- \leq \sup(x) + \sup x + \sup x \leq 3+$.

2.2 Definition [13, 14, 26]

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object taking the form $A= \{\langle x, T_A(x), I_A(x), F_A(x), \rangle : x \in X \}$, where $T_A(x):X \rightarrow [0,1]$, $I_A(x):X \rightarrow [0,1]$ and $F_A(x):X \rightarrow [0,1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively. For convenience, a SVN number is represented by $A= (a$, b, c$)$, where a, b, $c \in [0, 1]$ and $a+b+c \leq 3$.

2.3 Definition [14, 15, 16]

Suppose $\alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$, $\beta_{\tilde{a}} \in [0,1]$ and a_1 , a_2 , a_3 , $a_4 \in$ R, where $a_1 \leq a_2 \leq a_3 \leq a_4$. Then, a single valued trapezoidal neutrosophic number $\tilde{a}=\langle(a_1$, a_2 , a_3 , $a_4); \alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$, $\beta_{\tilde{a}}\rangle$ is a special neutrosophic set on the real line set R, whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right) & (a_1 \leq x \leq a_2) \\ \alpha_{\tilde{a}} & (a_2 \leq x \leq a_3) \\ \alpha_{\tilde{a}}\left(\frac{a_4-x}{a_4-a_3}\right) & (a_3 \leq x \leq a_4) \\ 0 & otherwise \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\theta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \leq x \leq a_2) \\ \alpha_{\tilde{a}} & (a_2 \leq x \leq a_3) \\ \frac{(x-a_3+\theta_{\tilde{a}}(a4-x))}{(a_4-a_3)} & (a_3 \leq x \leq a_4) \\ 1 & otherwise \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \leq x \leq a_2) \\ \alpha_{\tilde{a}} & (a_2 \leq x \leq a_3) \\ \frac{(x-a_3+\beta_{\tilde{a}}(a4-x))}{(a_4-a_3)} & (a_3 \leq x \leq a_4) \\ 1 & otherwise \end{cases} \quad (3)$$

where $\alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$ and $\beta_{\tilde{a}}$ represent the maximum truth-membership degree, the minimum indeterminacy-membership degree and the minimum falsity-membership degree, respectively. A single valued trapezoidal neutrosophic number $\tilde{a}=\langle(a_1$, a_2 , a_3 , $a_4); \alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$, $\beta_{\tilde{a}}\rangle$ may express an ill-defined quantity of the range, which is approximately equal to the interval $[a_2$, $a_3]$.

2.4 Definition [15, 14]

Let $\tilde{a}=\langle(a_1$, a_2 , a_3 , $a_4); \alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$, $\beta_{\tilde{a}}\rangle$ and $\tilde{b}=\langle(b_1$, b_2 , b_3 , $b_4); \alpha_{\tilde{b}}$, $\theta_{\tilde{b}}$, $\beta_{\tilde{b}}\rangle$ be two single valued trapezoidal neutrosophic numbers, and $\Upsilon \neq 0$ be any real number. Then:

- Addition of two trapezoidal neutrosophic numbers:

$$\tilde{a} + \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle$$

- Subtraction of two trapezoidal neutrosophic numbers:

$$\tilde{a} - \tilde{b} = \langle(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle$$

- Inverse of trapezoidal neutrosophic number:

$$\tilde{a}^{-1} = \left\langle \left(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \right\rangle \qquad \text{where } (\tilde{a} \neq 0)$$

- Multiplication of trapezoidal neutrosophic number by constant value:

$$\Upsilon \tilde{a} = \begin{cases} \langle (\Upsilon a_1, \Upsilon a_2, \Upsilon a_3, \Upsilon a_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle & \text{if } (\Upsilon > 0) \\ \langle (\Upsilon a_4, \Upsilon a_3, \Upsilon a_2, \Upsilon a_1); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle & \text{if } (\Upsilon < 0) \end{cases}$$

- Division of two trapezoidal neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_4 > 0, b_4 > 0) \\ \langle (\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_4 < 0, b_4 > 0) \\ \langle (\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

- Multiplication of trapezoidal neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_4 > 0, b_4 > 0) \\ \langle (a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_4 < 0, b_4 > 0) \\ \langle (a_4 b_4, a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

3 Methodology

In this study, we present the steps of the proposed model, we identify criteria, evaluate them, and decision makers also evaluate their judgments using neutrosophic trapezoidal numbers.

In previous articles, we noticed that the scale (1-9) has many drawbacks illustrated by [5]. We present a new scale from 0 to 1 to avoid this drawbacks. We use (n-1) judgments to obtain consistent trapezoidal neutrosophic preference relations instead of $\frac{n \times (n-1)}{2}$, in order to decrease the workload. ANP is used for ranking and selecting the alternatives.

The model of ANP in neutrosophic environment quantifies four criteria to combine them for decision making into one global variable. To do this, we first present the concept of ANP and determine the weight of each criterion based on opinions of decision makers.

Then, each alternative is evaluated with other criteria, considering the effects of relationships among criteria. The ANP technique is composed of four steps in the traditional way [17].

The steps of our ANP neutrosophic model can be introduced as:

Step - 1   constructing the model and problem structuring:

    1.   Selection of decision makers (DMs).

Form the problem in a network; the first level represents the goal and the second level represents criteria and sub-criteria and interdependence and feedback between criteria, and the third level represents the alternatives. An example of a network structure:
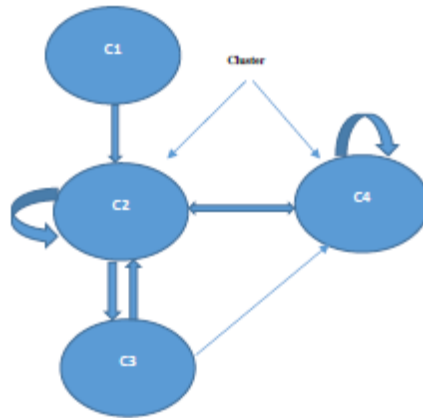

Figure 1. ANP model.

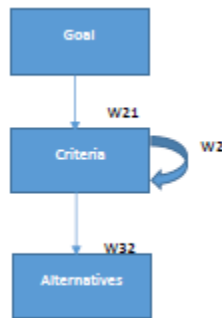Another example of a network ANP structure [17]:

Fig. 2. A Network Structure.

    2.   Prepare the consensus degree as it follows:

CD = $\frac{NE}{N}$ × 100%, where NE is the number of decision makers that have the same opinion and N is the total numbers of experts. Consensus degree should be greater than 50% [16].

Step - 2 Pairwise comparison matrices to determine weighting

1. Identify the alternatives of a problem A = {A1, A2, A3, …, Am}.

2. Identify the criteria and sub-criteria, and the interdependency between them:

   C = {C1, C2, C3, …, Cm}.

3. Determine the weighting matrix of criteria that is defined by decision makers (DMs) for each criterion (W_1).

4. Determine the relationship interdependencies among the criteria and the weights, the effect of each criterion against another in the range from 0 to 1.

5. Determine the interdependency matrix from multiplication of weighting matrix in step 3 and interdependency matrix in step 4.

6. Decision makers make pairwise comparisons matrix between alternatives compared to each criterion, and focus only on (n-1) consensus judgments instead of using $\frac{n \times (n-1)}{2}$ [16].

$$\tilde{R}= \begin{bmatrix} (l_{11}, m_{11l}, m_{11u}, u_{11}) & (l_{11}, m_{11l}, m_{11u}, u_{11}) & \cdots & (l_{1n}, m_{1nl}, m_{1nu}, u_{1n}) \\ (l_{21}, m_{21l}, m_{21u}, u_{21}) & (l_{22}, m_{22l}, m_{22u}, u_{22}) & \cdots & (l_{2n}, m_{2nl}, m_{2nu}, u_{2n}) \\ \cdots & \cdots & \cdots & \cdots \\ (l_{n1}, m_{n1l}, m_{n1u}, u_{n1}) & (l_{n2}, m_{n2l}, m_{n2u}, u_{n2}) & \cdots & (l_{nn}, m_{nnl}, m_{nnu}, u_{nn}) \end{bmatrix}$$

To make the comparisons matrix accepted, we should check the consistency of the matrix.

Definition 5 The consistency of a trapezoidal neutrosophic reciprocal preference relations $\tilde{R} = (\check{r}_{ij})$ n × n can be expressed as:

$\check{r}_{ij} = \check{r}_{ik} + \check{r}_{kj} - (0.5, 0.5, 0.5, 0.5)$ where i, j, k = 1, 2 … n. can also be written as $l_{ij} = l_{ik} + l_{kj} - (0.5, 0.5, 0.5, 0.5)$, $m_{ijl} = m_{ikl} + m_{mjl} - (0.5, 0.5, 0.5, 0.5)$, $m_{iju} = m_{iku} + m_{kju} - (0.5, 0.5, 0.5, 0.5)$, $u_{ij} = m_{ik} + m_{kj} - (0.5, 0.5, 0.5, 0.5)$, where i, j, k = 1, 2 … n and for $\check{r}_{ik} = 1- \check{r}_{kj}$ {Abdel-Basset, 2017 [16]}.

Definition 6 In order to check whether a trapezoidal neutrosophic reciprocal preference relation $\tilde{R}$ is additive approximation - consistency or not [16].

$$\check{r}_{ij} = \frac{\check{r}_{ij} + c_x}{1 + 2c_x} \tag{5}$$

$$\check{r}_{ij} = \frac{-\check{r}_{ij} + c_x}{1 + 2c_x} \tag{6}$$

$$u_{ij} - m_{ij} = \Delta \tag{7}$$

We transform the neutrosophic matrix to pairwise comparison deterministic matrix by adding (α, θ, β), and we use the following equation to calculate the accuracy and score

$$S(\tilde{a}_{ij}) = \frac{1}{16}[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}}) \tag{8}$$

and

$$A(\tilde{a}_{ij}) = \frac{1}{16}[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} + \beta_{\tilde{a}}) \tag{9}$$

We obtain the deterministic matrix by using $S(\tilde{a}_{ij})$.

From the deterministic matrix, we obtain the weighting matrix by dividing each entry by the sum of the column.

Step - 3 Formulation of supermatrix

The supermatrix concept is similar to the Markov chain process [18].

1. Determine scale and weighting data for the n alternatives against n criteria $w_{21}, w_{22}, w_{23}, \dots, w_{2n}$.

2. Determine the interdependence weighting matrix of criteria comparing it against another criteria in range from 0 to 1, defined as:

$$W_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_n \end{array} \begin{array}{cccc} C_1 & C_2 \quad C_3 & \quad C_n \\ \left[\begin{array}{cccc} (0-1) & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & (0-1) \end{array} \right] \end{array} \tag{10}$$

3. We obtain the weighting criteria $W_c = W_3 \times W_1$.

4. Determine the interdependence matrix $\tilde{A}_{criteria}$ among the alternatives with respect to each criterion.

$\tilde{A}_{criteria}$

$$\begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (l_{11}, m_{11l}, m_{11u}, u_{11}) & \cdots & (l_{1n}, m_{1nl}, m_{1nu}, u_{1n}) \\ (l_{21}, m_{21l}, m_{21u}, u_{21}) & (0.5, 0.5, 0.5, 0.5) & \cdots & (l_{2n}, m_{2nl}, m_{2nu}, u_{2n}) \\ \cdots & \cdots & (0.5, 0.5, 0.5, 0.5) & \cdots \\ (l_{n1}, m_{n1l}, m_{n1u}, u_{n1}) & (l_{n2}, m_{n2l}, m_{n2u}, u_{n2}) & \cdots & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

Step - 4 Selection of the best alternatives

1. Determine the priorities matrix of the alternatives with respect to each of the n criteria W_{An} where n is the number of criteria.

    Then, $W_{A1} = W_{\tilde{A}_{C1}} \times W_{21}$
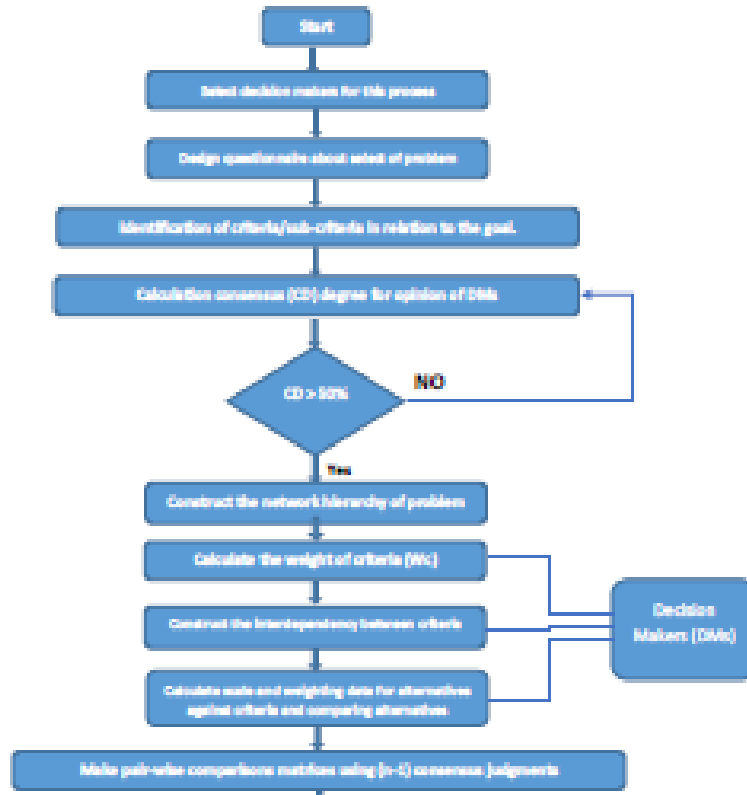
    $W_{A2} = W_{\tilde{A}_{C1}} \times W_{22}$

    $W_{A3} = W_{\tilde{A}_{C1}} \times W_{23}$

    $W_{An} = W_{\tilde{A}_{Cn}} \times W_{2n}$

    Then, $W_A = [W_{A1}, W_{A2}, W_{A3}, \dots, W_{An}]$.

2. In the last we rank the priorities of criteria and obtain the best alternatives by multiplication of the W_A matrix by the Weighting criteria matrix W_c, i.e.
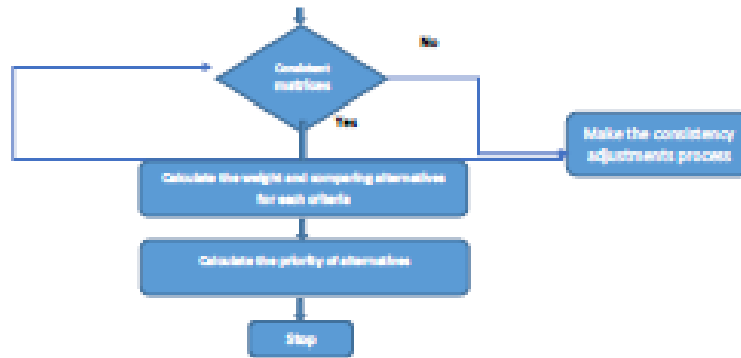
    $W_A \times W_c$

Figure 3. Schematic diagram of ANP with neutrosophic.

4 Numerical Example

In this section, we present an example to illustrate the ANP in neutrosophic environment - selecting the best personal car from four alternatives: Crossover is alternative A1, Sedan is alternative A2, Diesel is alternative A3, Nissan is alternative A4. We have four criteria C_j (j = 1, 2, 3, and 4), as follows: C_1 for price, C_2 for speed, C_3 for color, C_4 for model. The criteria to be considered is the supplier selections, which are determined by the DMs from a decision group. The team is split into four groups, namely DM_1, DM_2, DM_3 and DM_4, formed to select the most suitable alternatives. The criteria to be considered in the supplier's selection are determined by the DMs team from the expert's procurement office.
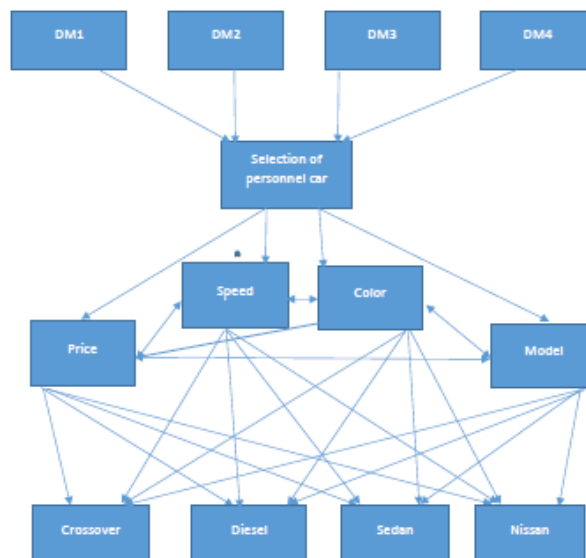

Figure 4. Network structure of the illustrative example.

In this example, we seek to illustrate the improvement and efficiency of ANP, the interdependency among criteria and feedback, and how a new scale from 0 to 1 improves and facilitates the solution and the ranking of the alternatives.

Step - 1: In order to compare the criteria, the decision makers assume that there is no interdependency among criteria. This data reflects relative weighting without considering interdependency among criteria. The weighting matrix of criteria that is defined by decision makers is $W_1 =$ (P, S, C, M) = (0.33, 0.40, 0.22, 0.05).

Step - 2: Assuming that there is no interdependency among the four alternatives, (A_1 A_2, A_3, A_4), they are compared against each criterion. Decision makers determine the relationships between each criterion and alternative, establishing the neutrosophic decision matrix between four alternatives (A_1, A_2, A_3, A_4) and four criteria (C_1, C_2 , C_3 , C_4):

$$R = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{bmatrix} C_1 & C_2 & C_3 & C_4 \\ (0.3,0.5,0.2,0.5) & (0.6,0.7,0.9,0.1) & (0.7,0.2,0.4,0.6) & (0.3,0.6,0.4,0.7) \\ (0.6,0.3,0.4,0.7) & (0.2,0.3,0.6,0.9) & (0.6,0.7,0.8,0.9) & (0.3,0.5,0.2,0.5) \\ (0.3,0.5,0.2,0.5) & (0.3,0.7,0.4,0.3) & (0.8,0.2,0.4,0.6) & (0.2,0.5,0.6,0.8) \\ (0.4,0.3,0.1,0.6) & (0.1,0.4,0.2,0.8) & (0.5,0.3,0.2,0.4) & (0.6,0.2,0.3,0.4) \end{bmatrix}$$

The last matrix appears consistent to definition 6 (5, 6, 7). Then, by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), minimum indeterminacy-membership degree (θ), and minimum falsity-membership degree (β) of single valued neutrosophic numbers, as in definition 6 (c). Therefore:

$$R = \begin{matrix} A1 \\ A2 \\ A3 \\ A3 \end{matrix} \begin{bmatrix} C_1 & C_2 & C_3 & C_4 \\ (0.3,0.5,0.2,0.5;0.3,0.4,0.6) & (0.6,0.7,0.9,0.1;0.4,0.3,0.5) & (0.7,0.2,0.4,0.6;0.8,0.4,0.2) & (0.3,0.6,0.4,0.7;0.4,0.5,0.6) \\ (0.6,0.3,0.4,0.7;0.2,0.5,0.8) & (0.2,0.3,0.6,0.9;0.6,0.2,0.5) & (0.6,0.7,0.8,0.9;0.2,0.5,0.7) & (0.3,0.5,0.2,0.5;0.5,0.7,0.8) \\ (0.3,0.5,0.2,0.5;0.4,0.5,0.7) & (0.3,0.7,0.4,0.3;0.2,0.5,0.9) & (0.8,0.2,0.4,0.6;0.4,0.6,0.5) & (0.2,0.5,0.6,0.8;0.4,0.3,0.8) \\ (0.4,0.3,0.1,0.6;0.2,0.3,0.5) & (0.1,0.4,0.2,0.8;0.7,0.3,0.6) & (0.5,0.3,0.2,0.4;0.3,0.4,0.7) & (0.6,0.2,0.3,0.4;0.6,0.3,0.4) \end{bmatrix}$$

$$S(\tilde{a}_{ij}) = \frac{1}{16}[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}})$$

And

$$A(\tilde{a}_{ij}) = \frac{1}{16}[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} + \beta_{\tilde{a}})$$

The deterministic matrix can be obtained by $S(\tilde{a}_{ij})$ equation in the following step:

$$R = \begin{matrix} A1 \\ A2 \\ A3 \\ A3 \end{matrix} \begin{matrix} C_1 & C_2 & C_3 & C_4 \\ \begin{bmatrix} 0.122 & 0.23 & 0.261 & 0.163 \\ 0.113 & 0.238 & 0.188 & 0.10 \\ 0.113 & 0.085 & 0.163 & 0.17 \\ 0.123 & 0.169 & 0.105 & 0.178 \end{bmatrix} \end{matrix}$$

Scale and weighting data for four alternatives against four criteria is derived by dividing each element by the sum of each column. The comparison matrix of four alternatives and four criteria is the following:

$$\begin{matrix} A1 \\ A2 \\ A3 \\ A3 \end{matrix} \begin{matrix} C_1 & C_2 & C_3 & C_4 \\ \begin{bmatrix} 0.259 & 0.319 & 0.364 & 0.268 \\ 0.240 & 0.329 & 0.262 & 0.164 \\ 0.240 & 0.118 & 0.227 & 0.278 \\ 0.261 & 0.234 & 0.146 & 0.291 \end{bmatrix} \\ w_{21} \quad w_{22} \quad w_{23} \quad w_{24} \end{matrix}$$

Step - 3: Decision makers take into consideration the interdependency among criteria. When one alternative is selected, more than one criterion should be considered. Therefore, the impact of all the criteria needs to be examined by using pairwise comparisons. By decision makers' group interviews, four sets of weightings have been obtained. The data that the decision makers prepare for the relationships between criteria reflect the relative impact degree of the four criteria with respect to each of four criteria. We make a graph to show the relationship between the interdependency among four criteria, and the mutual effect.
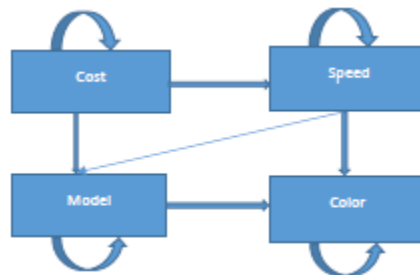

Figure 5. Interdependence among the criteria.

The interdependency weighting matrix of criteria is defined as:

$$
w_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ \begin{bmatrix} 1 & 0.8 & 0.4 & 0 \\ 0 & 0.2 & 0.5 & 0.6 \\ 0 & 0 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \end{array}
$$

$$
w_c = w_3 \times w_1 = \begin{bmatrix} 1 & 0.8 & 0.4 & 0 \\ 0 & 0.2 & 0.5 & 0.6 \\ 0 & 0 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \times \begin{bmatrix} 0.33 \\ 0.40 \\ 0.22 \\ 0.05 \end{bmatrix} = \begin{bmatrix} 0.738 \\ 0.220 \\ 0.037 \\ 0.005 \end{bmatrix}
$$

Thus, it is derived that $w_c = (C_1, C_2, C_3, C_4) = (0.738, 0.220, 0.037, 0.005)$.

Step - 4: The interdependency among alternatives with respect to each criterion is calculated by respect of consistency ratio that the decision makers determined. In order to satisfy the criteria 1 (C_1), which alternative contributes more to the action of alternative 1 against criteria 1 and how much more? We defined the project interdependency weighting matrix for criteria C_1 as:

a.   First criteria (C_1)
DMs compare criteria with other criteria, and determine the weighting of every criteria:

$$
\tilde{A}_{C1} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \\ \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.3,0.2,0.4,0.5) & y & y \\ y & (0.5,0.5,0.5,0.5) & (0.1,0.2,0.4,0.8) & y \\ y & y & (0.5,0.5,0.5,0.5) & (0.2,0.3,0.4,0.7) \\ y & y & y & (0.5,0.5,0.5,0.5) \end{bmatrix} \end{array}
$$

where y indicates preference values that are not determined by decision makers. Then, we can calculate these values and make them consistent with their judgments. Let us complete the previous matrix according to definition 5 as follows:

$\tilde{R}_{13} = \tilde{r}_{12} + \tilde{r}_{23} \ - (0.5,0.5,0.5,0.5) = (-0.1,-0.1,0.3,0.8)$

$\tilde{R}_{31} = 1 - \tilde{R}_{13} = 1 - (-0.1,-0.1,0.3,0.8) = (0.2,0.7,1.1,1.1)$

$\tilde{R}_{32} = \tilde{r}_{31} + \tilde{r}_{12} \ - (0.5,0.5,0.5,0.5) = (0.0,0.4,1.0,1.1)$

$\tilde{R}_{21} = 1 - \tilde{R}_{12} = 1 - (0.3,0.2,0.4,0.5) = (0.5,0.6,0.8,0.7)$

$\tilde{R}_{14} = \tilde{r}_{13} + \tilde{r}_{34} \ - (0.5,0.5,0.5,0.5) = (-0.1,-0.3,0.2,1.1)$

$\tilde{R}_{24} = \tilde{r}_{21} + \tilde{r}_{14} \ - (0.5,0.5,0.5,0.5) = (-0.1,-0.2,0.5,1.2)$

$\tilde{R}_{41} = 1 - \tilde{R}_{14} = 1 - (-0.1,-0.3,0.2,1.0) = (1.0,0.8,1.3,1.1)$

$\tilde{R}_{42} = 1 - \tilde{R}_{24} = 1 - (-0.1,-0.2,0.5,1.2) = (0.2,0.5,1.2,1.1)$

$\tilde{R}_{43} = 1 - \tilde{R}_{34} = 1 - (0.2,0.3,0.4,0.7) = (0.3,0.6,0.7,0.8)$

The comparison matrix will be as follows:

$$
\tilde{A}_{C1} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4 \\
\end{array}
$$

$$
\begin{bmatrix}
(0.5\,,0.5,0.5,0.5) & (0.3,0.2,0.4,0.5) & (-0.1,-0.1,0.3,0.8) & (-0.1,-0.3,0.2,1.1) \\
(0.5,0.6,0.8,0.7) & (0.5\,,0.5,0.5,0.5) & (0.1\,,0.2,0.4,0.8) & (-0.1,-0.2,0.5,1.2) \\
(0.2,0.7,1.1,1.1) & (0.0,0.4\,,1.0,1.1) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.3,0.4,0.7) \\
(1.0,0.8,1.3,1.1) & (0.2,0.5,1.2,1.1) & (0.3,0.6,0.7,0.8) & (0.5\,,0.5,0.5,0.5)
\end{bmatrix}
$$

According to definition 6, one can see that this relation is not a trapezoidal neutrosophic additive reciprocal preference relation. By using Eq. 5, Eq. 6 and Eq. 7 in definition 6, we obtain the following:

$$
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4 \\
\end{array}
$$

$$
\tilde{A}_{C1} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{bmatrix}
(0.5\,,0.5,0.5,0.5) & (0.3,0.2,0.4,0.5) & (0.1,0.1,0.3,0.8) & (0.1,0.3,0.2,1.0) \\
(0.5,0.6,0.8,0.7) & (0.5\,,0.5,0.5,0.5) & (0.1\,,0.2,0.4,0.8) & (0.1,0.2,0.5,1.0) \\
(0.2,0.7,1.0,1.0) & (0.0,0.4\,,1.0,1.0) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.3,0.4,0.7) \\
(1.0,0.8,1.0,1.0) & (0.2,0.5,1.0,1.0) & (0.3,0.6,0.7,0.8) & (0.5\,,0.5,0.5,0.5)
\end{bmatrix}
$$

We check if the matrix is consistent according to definition 6. By ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), the minimum indeterminacy-membership degree (θ) and the minimum falsity-membership degree (β) of single valued neutrosophic numbers as in definition 6.

$$
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4 \\
\end{array}
$$

$$
\tilde{A}_{C1} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
$$

$$
\begin{bmatrix}
(0.5\,,0.5,0.5,0.5) & (0.3,0.2,0.4,0.5;0.7\,,0.2,0.5) & (0.1,0.1,0.3,0.8;0.5\,,0.2,0.1) & (0.1,0.3,0.2,1.0;0.5,0.2,0.1) \\
(0.5,0.6,0.8,0.7;0.7\,,0.2,0.5) & (0.5\,,0.5,0.5,0.5) & (0.1\,,0.2,0.4,0.8;0.4,0.5,0.6) & (0.1,0.2,0.5,1.0;0.5,0.1,0.2) \\
(0.2,0.7,1.0,1.0;0.8,0.2\,,0.1) & (0.0,0.4\,,1.0,1.0;0.3,0.1,0.5) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.3,0.4,0.7;0.7,0.2,0.5) \\
(1.0,0.8,1.0,1.0;0.6,0.2,0.3) & (0.2,0.5,1.0,1.0;0.6,0.2,0.3) & (0.3,0.6,0.7,0.8;0.9,0.4,0.6) & (0.5\,,0.5,0.5,0.5)
\end{bmatrix}
$$

We make sure the matrix is deterministic, or we transform the previous matrix to be a deterministic pairwise comparison matrix, to calculate the weight of each criterion using equation (8, 9) in definition 6.

The deterministic matrix can be obtained by S $(\tilde{a}_{ij})$ equation in the following step:

$$
\tilde{A}_{C1} = \begin{bmatrix}
0.5 & 0.175 & 0.179 & 0.22 \\
0.325 & 0.5 & 0.122 & 0.25 \\
0.453 & 0.265 & 0.5 & 0.2 \\
0.38 & 0.354 & 0.285 & 0.5
\end{bmatrix}
$$

We present the weight of each alternatives according to each criteria from the deterministic matrix easily by dividing each entry by the sum of the column; we obtain the following matrix as:

$$\tilde{A}_{C1} = \begin{bmatrix} 0.30 & 0.135 & 0.165 & 0.188 \\ 0.196 & 0.386 & 0.112 & 0.214 \\ 0.273 & 0.198 & 0.460 & 0.171 \\ 0.229 & 0.274 & 0.262 & 0.427 \end{bmatrix}$$

b. Second criteria (C_2)

DMs compare criteria with other criteria, and determine the weighting of every criteria:

$$\tilde{A}_{C2} = \begin{matrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.3,0.6,0.4,0.5) & y & y \\ y & (0.5,0.5,0.5,0.5) & (0.5,0.2,0.4,0.9) & y \\ y & y & (0.5,0.5,0.5,0.5) & (0.5,0.3,0.4,0.7) \\ y & y & y & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

where y indicates preference values that are not determined by decision makers, then we can calculate these values and make them consistent with their judgments.

We complete the previous matrix according to definition 5 as follows:

The comparison matrix will be as follows:

$$\tilde{A}_{C2} = \begin{matrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.3,0.6,0.4,0.5) & (0.3,0.3,0.3,0.9) & (0.3,0.1,0.2,1.1) \\ (0.5,0.6,0.4,0.7) & (0.5,0.5,0.5,0.5) & (0.5,0.2,0.4,0.9) & (0.3,0.2,0.1,1.3) \\ (0.1,0.7,0.7,0.7) & (-0.1,0.8,0.3,0.5) & (0.5,0.5,0.5,0.5) & (0.5,0.3,0.4,0.7) \\ (1.0,0.8,0.9,0.7) & (0.3,0.9,0.8,0.7) & (0.3,0.6,0.7,0.5) & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

According to definition 6, one can see that this relation is not a trapezoidal neutrosophic additive reciprocal preference relation. By using Eq. 5, Eq. 6 and Eq. 7 in definition 6, we obtain the following:

$$\tilde{A}_{C2} = \begin{matrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.3,0.6,0.4,0.5) & (0.3,0.3,0.3,0.9) & (0.3,0.1,0.2,1.0) \\ (0.5,0.6,0.4,0.7) & (0.5,0.5,0.5,0.5) & (0.5,0.2,0.4,0.9) & (0.3,0.2,0.1,1.0) \\ (0.1,0.7,0.7,0.7) & (0.1,0.8,0.3,0.5) & (0.5,0.5,0.5,0.5) & (0.5,0.3,0.4,0.7) \\ (1.0,0.8,0.9,0.7) & (0.3,0.9,0.8,0.7) & (0.3,0.6,0.7,0.5) & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

Let us check that the matrix is consistent according to definition 6. Then, by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference

relations, decision makers (DMs) should determine the maximum truth-membership degree (α), the minimum indeterminacy-membership degree (θ) and the minimum falsity-membership degree (β) of single valued neutrosophic numbers, as in definition 6. Then:

$$
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4
\end{array}
$$

$$
\tilde{A}_{C2} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
$$

$$
\begin{bmatrix}
(0.5\,,0.5,0.5,0.5) & (0.3,0.6,0.4,0.5\,;0.7\,,0.3,0.5) & (0.3,0.3,0.3,0.9\,;0.5,0.2,0.1) & (0.3,0.1,0.2,1.0;0.5,0.2,0.1) \\
(0.5,0.6,0.4,0.7;0.7\,,0.3,0.5) & (0.5\,,0.5,0.5,0.5) & (0.5\,,0.2,0.4,0.9;0.4,0.5,0.6) & (0.3,0.2,0.1,1.0;0.5,0.1,0.4) \\
(0.1,0.7,0.7,0.7;0.8,0.2\,,0.3) & (0.1,0.8\,,0.3,0.5;0.4,0.2,0.5) & (0.5\,,0.5,0.5,0.5) & (0.5\,,0.3,0.4,0.7;0.6,0.2,0.5) \\
(1.0,0.8,0.9,0.7;0.6,0.4,0.3) & (0.3,0.9,0.8,0.7;0.6,0.2,0.3) & (0.3,0.6,0.7,0.5;0.9,0.4,0.5) & (0.5\,,0.5,0.5,0.5)
\end{bmatrix}
$$

Let us be sure the matrix is deterministic, or transform the previous matrix to be deterministic pairwise comparison matrix, to calculate the weight of each criteria using equation (8, 9) in definition 6.

The deterministic matrix can be obtained by S ($\tilde{a}_{ij}$) equation in the following step:

$$
\tilde{A}_{C2} = \begin{bmatrix}
0.5 & 0.214 & 0.247 & 0.22 \\
0.216 & 0.5 & 0.163 & 0.20 \\
0.316 & 0.181 & 0.5 & 0.226 \\
0.404 & 0.354 & 0.3 & 0.5
\end{bmatrix}
$$

We present the weight of each alternatives according to each criteria from the deterministic matrix by dividing each entry by the sum of the column; we obtain the following matrix:

$$
\tilde{A}_{C2} = \begin{bmatrix}
0.50 & 0.215 & 0.244 & 0.192 \\
0.216 & 0.503 & 0.161 & 0.175 \\
0.273 & 0.182 & 0.495 & 0.197 \\
0.229 & 0.356 & 0.259 & 0.436
\end{bmatrix}
$$

c.   Third criteria (C_3)

DMs compare criteria with other criteria, and determine the weight of every criteria.

$$
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4
\end{array}
$$

$$
\tilde{A}_{C3} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{bmatrix}
(0.5\,,0.5,0.5,0.5) & (0.6,0.7,0.9,0.1) & y & y \\
y & (0.5\,,0.5,0.5,0.5) & (0.6\,,0.7,0.8,0.9) & y \\
y & y & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.5,0.6,0.8) \\
y & y & y & (0.5\,,0.5,0.5,0.5)
\end{bmatrix}
$$

where y indicates preference values that are not determined by decision makers; then, we can calculate these values and make them consistent with their judgments.

We complete the previous matrix according to definition 5 as follows:

$$
\begin{array}{cccc}
& A_1 & A_2 & A_3 & A_4
\end{array}
$$

$$
\tilde{A}_{C3} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{bmatrix}
(0.5,0.5,0.5,0.5) & (0.6,0.7,0.9,0.1) & (0.7,0.9,1.2,1.4) & (0.4,0.7,1.3,1.7) \\
(0.0,0.1,0.3,0.4) & (0.5,0.5,0.5,0.5) & (0.6,0.7,0.8,0.9) & (0.3,0.5,0.9,1.2) \\
(-0.4,-0.2,0.1,0.3) & (-0.3,0.0,0.5,0.8) & (0.5,0.5,0.5,0.5) & (0.2,0.5,0.6,0.8) \\
(-0.7,-0.3,0.3,0.6) & (-0.6,-0.1,0.7,1.1) & (0.2,0.4,0.5,0.8) & (0.5,0.5,0.5,0.5)
\end{bmatrix}
$$

According to definition 6, one can see that the relation is not a trapezoidal neutrosophic additive reciprocal preference relation. By using Eq. 5, Eq. 6 and Eq. 7 in definition 6, we obtain the following:

$$
\begin{array}{cccc}
& A_1 & A_2 & A_3 & A_4
\end{array}
$$

$$
\tilde{A}_{C3} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{bmatrix}
(0.5,0.5,0.5,0.5) & (0.6,0.7,0.9,0.1) & (0.7,0.9,1.0,1.0) & (0.4,0.7,1.0,1.0) \\
(0.0,0.1,0.3,0.4) & (0.5,0.5,0.5,0.5) & (0.6,0.7,0.8,0.9) & (0.3,0.5,0.9,1.0) \\
(0.4,0.2,0.1,0.3) & (0.3,0.0,0.5,0.8) & (0.5,0.5,0.5,0.5) & (0.2,0.5,0.6,0.8) \\
(0.7,0.3,0.3,0.6) & (0.6,0.1,0.7,1.0) & (0.2,0.4,0.5,0.8) & (0.5,0.5,0.5,0.5)
\end{bmatrix}
$$

Then, let us check that the matrix is consistent according to definition 6. Then, by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), the minimum indeterminacy-membership degree (θ) and the minimum falsity-membership degree (β) of the single valued neutrosophic numbers as in definition 6. Then:

$$
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4
\end{array}
$$

$$
\tilde{A}_{C3} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
$$

$$
\begin{bmatrix}
(0.5,0.5,0.5,0.5) & (0.6,0.7,0.9,0.1;0.7,0.2,0.5) & (0.7,0.9,1.0,1.0;0.5,0.2,0.1) & (0.4,0.7,1.0,1.0;0.5,0.2,0.3) \\
(0.0,0.1,0.3,0.4;0.8,0.2,0.6) & (0.5,0.5,0.5,0.5) & (0.6,0.7,0.8,0.9;0.5,0.2,0.1) & (0.3,0.5,0.9,1.0;0.5,0.1,0.2) \\
(0.4,0.2,0.1,0.3;0.5,0.3,0.4) & (0.3,0.0,0.5,0.8;0.8,0.5,0.3) & (0.5,0.5,0.5,0.5) & (0.2,0.5,0.6,0.8;0.6,0.4,0.2) \\
(0.7,0.3,0.3,0.6;0.5,0.2,0.1) & (0.6,0.1,0.7,1.0;0.3,0.1,0.5) & (0.2,0.4,0.5,0.8;0.3,0.1,0.5) & (0.5,0.5,0.5,0.5)
\end{bmatrix}
$$

Let us be sure the matrix is deterministic, or transform the previous matrix to be deterministic pairwise comparison matrix, to calculate the weight of each criteria using equation (8, 9) in definition 6.

The deterministic matrix can be obtained by S ($\tilde{a}_{ij}$) equation in the following step:

$$
\tilde{A}_{C3} = \begin{bmatrix}
0.5 & 0.4 & 0.49 & 0.41 \\
0.1 & 0.5 & 0.41 & 0.37 \\
0.18 & 0.24 & 0.5 & 0.56 \\
0.38 & 0.30 & 0.20 & 0.5
\end{bmatrix}
$$

We present the weight of each alternatives according to each criteria from the deterministic matrix by dividing each entry by the sum of the column; we obtain the following matrix:

$$\tilde{A}_{C3} = \begin{bmatrix} 0.43 & 0.27 & 0.30 & 0.22 \\ 0.08 & 0.35 & 0.26 & 0.20 \\ 0.15 & 0.16 & 0.31 & 0.30 \\ 0.33 & 0.21 & 0.12 & 0.27 \end{bmatrix}$$

d. Four criteria (C_4)

DMs compare criteria with other criteria, and determine the weighting of every:

$$\tilde{A}_{C4} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.4,0.5,0.3,0.7) & y & y \\ y & (0.5,0.5,0.5,0.5) & (0.4,0.2,0.7,0.5) & y \\ y & y & (0.5,0.5,0.5,0.5) & (0.4,0.6,0.5,0.8) \\ y & y & y & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

Where y indicates the preference values that are not determined by decision makers; then, we can calculate these values and make them consistent with their judgments.

We complete the previous matrix according to definition 5 as follows:

$$\tilde{A}_{C4} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.4,0.5,0.3,0.7) & (0.3,0.2,0.5,0.7) & (0.2,0.3,0.5,1.0) \\ (0.3,0.7,0.5,0.6) & (0.5,0.5,0.5,0.5) & (0.4,0.2,0.7,0.5) & (0.0,0.5,0.5,1.1) \\ (0.3,0.7,0.5,0.6) & (0.2,0.5,0.6,0.9) & (0.5,0.5,0.5,0.5) & (0.4,0.6,0.5,0.8) \\ (0.3,0.7,0.5,0.6) & (-0.1,0.5,0.5,1.0) & (0.2,0.5,0.4,0.6) & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

According to definition 6, one can see that this relation is not a trapezoidal neutrosophic additive reciprocal preference relation. By using Eq. 5, Eq. 6 and Eq. 7 in definition 6, we obtain the following:

$$\tilde{A}_{C4} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.4,0.5,0.3,0.7) & (0.3,0.2,0.5,0.7) & (0.2,0.3,0.5,1.0) \\ (0.3,0.7,0.5,0.6) & (0.5,0.5,0.5,0.5) & (0.4,0.2,0.7,0.5) & (0.0,0.5,0.5,1.0) \\ (0.3,0.7,0.5,0.6) & (0.2,0.5,0.6,0.9) & (0.5,0.5,0.5,0.5) & (0.4,0.6,0.5,0.8) \\ (0.3,0.7,0.5,0.6) & (0.1,0.5,0.5,1.0) & (0.2,0.5,0.4,0.6) & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

Then, we check that the matrix is consistent according to definition 6. By ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), the minimum indeterminacy-membership degree (θ) and the minimum falsity-membership degree (β) of the single valued neutrosophic numbers, as in definition 6.

$$
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4
\end{array}
$$

$$
\tilde{A}_{C4} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
$$

$$
\begin{bmatrix}
(0.5, 0.5, 0.5, 0.5) & (0.4, 0.5, 0.3, 0.7\,;0.4\,, 0.3, 0.6) & (0.3, 0.2, 0.5, 0.7\,;0.2\,, 0.3, 0.5) & (0.2, 0.3, 0.5, 1.0; 0.3, 0.1, 0.8) \\
(0.3, 0.7, 0.5, 0.6; 0.7\,, 0.4, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.4\,, 0.2, 0.7, 0.5; 0.3, 0.5, 0.6) & (0.0, 0.5, 0.5, 1.0; 0.4, 0.3, 0.2) \\
(0.3, 0.5, 0.8, 0.7; 0.7\,, 0.4, 0.5) & (0.2, 0.5\,, 0.6, 0.9; 0.7, 0.4, 0.3) & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.6, 0.5, 0.8; 0.7, 0.3, 0.5) \\
(0.0, 0.5, 0.7, 0.8; 0.5, 0.2, 0.4) & (0.1, 0.5, 0.5, 1.0; 0.5, 0.3, 0.6) & (0.2, 0.5, 0.4, 0.6; 0.4, 0.6, 0.2) & (0.5, 0.5, 0.5, 0.5)
\end{bmatrix}
$$

Let us be sure the matrix is deterministic, or transform the previous matrix to be deterministic pairwise comparison matrix, to calculate the weight of each criteria using equation (8, 9) in definition 6.

The deterministic matrix can be obtained by S $(\tilde{a}_{ij})$ equation in the following step:

$$
\tilde{A}_{C4} = \begin{bmatrix}
0.5 & 0.18 & 0.15 & 0.17 \\
0.24 & 0.5 & 0.13 & 0.23 \\
0.29 & 0.27 & 0.5 & 0.27 \\
0.23 & 0.21 & 0.17 & 0.5
\end{bmatrix}
$$

We present the weight of each alternative according to each criteria from the deterministic matrix by dividing each entry by the sum of the column; we obtain the following matrix:

$$
\tilde{A}_{C4} = \begin{bmatrix}
0.40 & 0.16 & 0.16 & 0.15 \\
0.19 & 0.43 & 0.14 & 0.19 \\
0.23 & 0.23 & 0.5 & 0.23 \\
0.18 & 0.18 & 0.18 & 0.42
\end{bmatrix}
$$

Step 4: The priorities of the alternative W_A with respect to each of the four criteria are given by synthesizing the results from Steps 2 and 4 as follows:

$$
W_{A1} = W_{\tilde{A}_{C1}} \times W_{21} = \begin{bmatrix} 0.199 \\ 0.172 \\ 0.273 \\ 0.299 \end{bmatrix}
$$

$$
W_{A2} = W_{\tilde{A}_{C2}} \times W_{22} = \begin{bmatrix} 0.303 \\ 0.294 \\ 0.251 \\ 0.347 \end{bmatrix}
$$

$$
W_{A3} = W_{\tilde{A}_{C3}} \times W_{23} = \begin{bmatrix} 0.327 \\ 0.209 \\ 0.210 \\ 0.241 \end{bmatrix}
$$

$$
W_{A4} = W_{\tilde{A}_{C4}} \times W_{24} = \begin{bmatrix} 0.222 \\ 0.216 \\ 0.305 \\ 0.250 \end{bmatrix}
$$

The matrix W_A is defined by grouping together the above four columns:

$$
W_A = [\, W_{A1}, W_{A2}, W_{A3}, W_{A4} \,]
$$

Step 5: The overall priorities for the candidate alternatives are finally calculated by multiplying W_A and W_c:

$$= W_A \times W_c = \begin{matrix} W_{A1} & W_{A2} & W_{A3} & W_{A4} \\ \begin{bmatrix} 0.199 & 0.303 & 0.327 & 0.222 \\ 0.172 & 0.294 & 0.209 & 0.216 \\ 0.273 & 0.251 & 0.210 & 0.305 \\ 0.299 & 0.347 & 0.241 & 0.250 \end{bmatrix} \end{matrix} \times \begin{bmatrix} 0.738 \\ 0.220 \\ 0.037 \\ 0.005 \end{bmatrix} = \begin{bmatrix} 0.226 \\ 0.200 \\ 0.265 \\ 0.307 \end{bmatrix}$$

The final results in the ANP Neutrosophic Phase are (A1, A2, A3, A4) = (0.226, 0.200, 0.265, 0.307). These ANP Neutrosophic results are interpreted as follows. The highest weighting of criteria in this problem selection example is A4. Next is A1. These weightings are used as priorities in selecting the best personnel car.

Then, it is obvious that the four alternative has the highest rank, meaning that Nissan is the best car according to this criteria, followed by Crossover, Diesel and, finally, Sedan.

Table 2.   Ranking of alternatives.

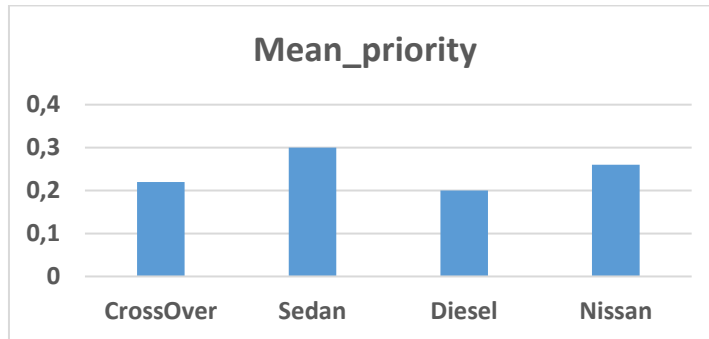| Car Name	Priority
Crossover	0.22
Diesel	0.20
Nissan	0.26
Sedan	0.30

Figure 6. ANP ranking of alternatives.

5 Conclusion

This research employed the ANP technique in neutrosophic environment for solving complex problems, showing the interdependence among criteria, the feedback and the relative weight of decision makers (DMs). We analyzed how to determine the weight for each criterion, and the interdependence among criteria,

calculating the weighting of each criterion to each alternative. The proposed model of ANP in neutrosophic environment is based on using of $(n-1)$ consensus judgments instead of $\frac{n \times (n-1)}{2}$ ones, in order to decrease the workload. We used a new scale from 0 to 1 instead of that from 1 to 9. We also presented a real life example as a case study. In the future, we plan to apply ANP in neutrosophic environment by integrating it with other techniques, such as TOPSIS.

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V

Interval Complex Neutrosophic Graph of Type 1

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Abstract

The neutrosophic set theory, proposed by smarandache, can be used as a general mathematical tool for dealing with indeterminate and inconsistent information. By applying the concept of neutrosophic sets on graph theory, several studies of neutrosophic models have been presented in the literature. In this paper, the concept of complex neutrosophic graph of type 1 is extended to interval complex neutrosophic graph of type 1(ICNG1). We have proposed a representation of ICNG1 by adjacency matrix and studied some properties related to this new structure. The concept of ICNG1 generalized the concept of generalized fuzzy graphs of type 1 (GFG1), generalized single valued neutrosophic graphs of type 1 (GSVNG1) generalized interval valued neutrosophic graphs of type 1 (GIVNG1) and complex neutrosophic graph type 1(CNG1).

Keywords

Neutrosophic set; complex neutrosophic set; interval complex neutrosophic set; interval complex neutrosophic graph of type 1; adjacency matrix.

1 Introduction

Crisp set, fuzzy sets [14] and intuitionisitic fuzzy sets [13] already acts as a mathematical tool. But Smarandache [5, 6] gave a momentum by introducing

the concept of neutrosophic sets (NSs in short). Neutrosophic sets came as a glitter in this field as their vast potential to intimate imprecise, incomplete, uncertainty and inconsistent information of the world. Neutrosophic sets associates a degree of membership (T) , indeterminacy(I) and non- membership (F) for an element each of which belongs to the non-standard unit interval]−0, 1+[. Due to this characteristics, the practical implement of NSs becomes difficult. So, for this reason, Smarandache [5, 6] and Wang et al. [10] introduced the concept of a single valued Neutrosophic sets (SVNS), which is an instance of a NS and can be used in real scientific and engineering applications. Wang et al. [12] defined the concept of interval valued neutrosophic sets as generalization of SVNS. In [11], the readers can found a rich literature on single valued neutrosophic sets and their applications in divers fields.

Graph representations are widely used for dealing with structural information, in different domains such as networks, image interpretation, pattern recognition operations research. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes values from [0, 1].In [1] Atanassov defined the concept of intuitionistic fuzzy graphs (IFGs) with vertex sets and edge sets as IFS. The concept of fuzzy graphs and their extensions have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects.

Fuzzy graphs and their extensions such as hesitant fuzzy graph, intuitionistic fuzzy graphs ..etc , deal with the kinds of real life problems having some uncertainty measure. All these graphs cannot handle the indeterminate relationship between object. So, for this reason, Smaranadache [3,9]defined a new form of graph theory called neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. The same author[4]initiated a new graphical structure of neutrosphic graphs based on (T, I, F) components and proposed three structures of neutrosophic graphs such as neutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. In [8] Smarandache defined a new classes of neutrosophic graphs including neutrosophic offgraph, neutrosophic bipolar/tripola/ multipolar graph. Single valued neutrosphic graphs with vertex sets and edge sets as SVN were first introduced by Broumi [33] and defined some of its properties. Also, Broumi et al.[34] defined certain degrees of SVNG and established some of their properties. The same author proved a necessary and sufficient condition for a single valued neutrosophic graph to be an isolated-SVNG [35]. In addition, Broumi et al. [47] defined the concept of the interval valued neutrosophic graph as a generalization of SVNG and analyzed some properties of it. Recently, Several extension of

single valued neutrosophic graphs, interval valued neutrosophic graphs and their application have been studied deeply [17-19, 21-22, 36-45, 48-49,54-56].

In [7] Smarandache initiated the idea of removal of the edge degree restriction of fuzzy graphs, intuitionistic fuzzy graphs and single valued neutrosophic graphs. Samanta et al [53] discussed the concept of generalized fuzzy graphs (GFG) and studied some properties of it . The authors claim that fuzzy graphs and their extension defined by many researches are limited to represented for some systems such as social network. Employing the idea initiated by smarandache [7], Broumi et al. [46, 50,51]proposed a new structures of neutrosophic graphs such as generalized single valued neutrosophic graph of type1(GSVNG1), generalized interval valued neutrosophic graph of type1(GIVNG1), generalized bipolar neutrosophic graph of type 1, all these types of graphs are a generalization of generalized fuzzy graph of type1[53]. In [2], Ramot defined the concept of complex fuzzy sets as an extension of the fuzzy set in which the range of the membership function is extended from the subset of the real number to the unit disc. Later on, some extensions of complex fuzzy set have been studied well in the litteratur e[20,23,26,28,29,58-68].In [15],Ali and Smarandache proposed the concept of complex neutrosophic set in short CNS. The concept of complex neutrosophic set is an extension of complex intuitionistic fuzzy sets by adding by adding complex-valued indeterminate membership grade to the definition of complex intuitionistic fuzzy set. The complex-valued truth membership function, complex-valued indeterminacy membership function, and complex-valued falsity membership function are totally independent. The complex fuzzy set has only one extra phase term, complex intuitionistic fuzzy set has two additional phase terms while complex neutrosophic set has three phase terms. The complex neutrosophic sets (CNS) are used to handle the information of uncertainty and periodicity simultaneously. When the values of the membership function indeterminacy-membership function and the falsity-membership function in a CNS are difficult to be expressed as exact single value in many real-world problems, interval complex neutrosophic sets can be used to characterize the uncertain information more sufficiently and accurately. So for this purpose, Ali et al [16] defined the concept of interval complex neutrosophic sets (ICNs) and examined its characteristics. Recently, Broumi et al.[52]defined the concept of complex neutrosophic graphs of type 1 with vertex sets and edge sets as complex neutrosophic sets.

In this paper, an extended version of complex neutrosophic graph of type 1(ICNG1) is introduced. To the best of our knowledge, there is no research on interval complex neutrosophic graph of type 1 in literature at present.

The remainder of this paper is organized as follows. In Section 2, some fundamental and basic concepts regarding neutrosophic sets, single valued neutrosophic sets, complex neutrosophic set, interval complex neutrosophic set and complex neutrosophic graphs of type 1 are presented. In Section 3, ICNG1 is proposed and provided by a numerical example. In section 4 a representation matrix of ICNG1 is introduced and finally we draw conclusions in section 5.

2 Fundamental and Basic Concepts

In this section we give some definitions regarding neutrosophic sets, single valued neutrosophic sets, complex neutrosophic set, interval complex neutrosophic set and complex neutrosophic graphs of type 1

Definition 2.1 [5, 6]

Let ζ be a space of points and let x $\in \zeta$. A neutrosophic set A $\in \zeta$ is characterized by a truth membership function T, an indeterminacy membership function I, and a falsity membership function F. The values of T, I, F are real standard or nonstandard subsets of $]^-0,1^+[$, and T, I, F: $\zeta \rightarrow]^-0,1^+[$. A neutrosophic set can therefore be represented as

$$A=\{(x, T_A(x), I_A(x), F_A(x)): x \in \zeta\} \quad (1)$$

Since $T, I, F \in [0, 1]$, the only restriction on the sum of T, I, F is as given below:

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (2)$$

From philosophical point of view, the NS takes on value from real standard or non-standard subsets of $]^-0,1^+[$. However, to deal with real life applications such as engineering and scientific problems, it is necessary to take values from the interval [0, 1] instead of $]^-0,1^+[$.

Definition 2.2 [10]

Let ζ be a space of points (objects) with generic elements in ζ denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in ζ, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. The SVNS A can therefore be written as

$$A=\{(x, T_A(x), I_A(x), F_A(x)): x \in \zeta\} \quad (3)$$

Definition 2.3 [15]

A complex neutrosophic set A defined on a universe of discourse X, which is characterized by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ that assigns a complex-valued membership grade to $T_A(x), I_A(x), F_A(x)$ for any $x \in X$. The values of $T_A(x), I_A(x), F_A(x)$ and their sum may be any values within a unit circle

in the complex plane and is therefore of the form $T_A(x) = p_A(x)e^{i\mu_A(x)}$, $I_A(x) = q_A(x)e^{iv_A(x)}$, and $F_A(x) = r_A(x)e^{i\omega_A(x)}$. All the amplitude and phase terms are real-valued and $p_A(x), q_A(x), r_A(x) \in [0,1]$, whereas $\mu_A(x), v_A(x), \omega_A(x) \in (0, 2\pi]$, such that the condition.

$$0 \le p_A(x) + q_A(x) + r_A(x) \le 3 \tag{4}$$

is satisfied. A complex neutrosophic set A can thus be represented in set form as:

$$A = \{\langle x, T_A(x) = a_T, I_A(x) = a_I, F_A(x) = a_F \rangle : x \in X\}, \tag{5}$$

Where $T_A: X \to \{a_T : a_T \in C, |a_T| \le 1\}, I_A: X \to \{a_I : a_I \in C, |a_I| \le 1\}, F_A: X \to \{a_F : a_F \in C, |a_F| \le 1\}$, and also

$$|T_A(x) + I_A(x) + F_A(x)| \le 3. \tag{6}$$

Let A and B be two CNSs in X, which are as defined as follow $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ and $B = \{(x, T_B(x), I_B(x), F_B(x)) : x \in X\}$.

Definition 2.4 [15]

Let A and B be two CNSs in X. The union, intersection and complement of two CNSs are defined as:

The union of A and B denoted as $A \cup_N B$, is defined as:

$$A \cup_N B = \{(x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)) : x \in X\}, \tag{7}$$

Where, $T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)$ are given by

$$T_{A\cup B}(x) = \max(p_A(x), p_B(x)) . e^{i\,(\mu_A(x)\cup\mu_B(x))}$$

$$I_{A\cup B}(x) = \min(q_A(x), q_B(x)) . e^{i\,(v_A(x)\cup v_B(x))},$$

$$F_{A\cup B}(x) = \min(r_A(x), r_B(x)) . e^{i\,(\omega_A(x)\cup\omega_B(x))}.$$

The intersection of A and B denoted as $A \cap_N B$, is defined as:

$$A \cap_N B = \{(x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)) : x \in X\}, \tag{8}$$

Where $T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)$ are given by

$$T_{A\cup B}(x) = \min(p_A(x), p_B(x)) . e^{i\,(\mu_A(x)\cap\mu_B(x))} \tag{9}$$

$$I_{A\cup B}(x) = \max(q_A(x), q_B(x)) . e^{i\,(v_A(x)\cap v_B(x))}, \tag{10}$$

$$F_{A\cup B}(x) = \max(r_A(x), r_B(x)) . e^{i\,(\omega_A(x)\cap\omega_B(x))}. \tag{11}$$

The union and the intersection of the phase terms of the complex truth, falsity and indeterminacy membership functions can be calculated using any one of the following operations:

Sum:

$$\mu_{A\cup B}(x) = \mu_A(x) + \mu_B(x), \tag{12}$$

$$v_{A\cup B}(x) = v_A(x) + v_B(x), \tag{13}$$

$$\omega_{A\cup B}(x) = \omega_A(x) + \omega_B(x). \tag{14}$$

Max:

$$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)), \tag{15}$$

$$\nu_{A\cup B}(x) = \max(\nu_A(x), \nu_B(x)), \tag{16}$$

$$\omega_{A\cup B}(x) = \max(\omega_A(x), \omega_B(x)). \tag{17}$$

Min:

$$\mu_{A\cup B}(x) = \min(\mu_A(x), \mu_B(x)), \tag{18}$$

$$\nu_{A\cup B}(x) = \min(\nu_A(x), \nu_B(x)), \tag{19}$$

$$\omega_{A\cup B}(x) = \min(\omega_A(x), \omega_B(x)). \tag{20}$$

"The game of winner, neutral, and loser":

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & if \quad p_A > p_B \\ \mu_B(x) & if \quad p_B > p_A \end{cases}, \tag{21}$$

$$\nu_{A\cup B}(x) = \begin{cases} \nu_A(x) & if \quad q_A < q_B \\ \nu_B(x) & if \quad q_B < q_A \end{cases}, \tag{22}$$

$$\omega_{A\cup B}(x) = \begin{cases} \omega_A(x) & if \quad r_A < r_B \\ \omega_B(x) & if \quad r_B < r_A \end{cases}. \tag{23}$$

Definition 2.5 [16]

An interval complex neutrosophic set A defined on a universe of discourse ζ, which is characterized by an interval truth membership function $\tilde{T}_A(x) = [T_A^L(x), T_A^U(x)]$, an interval indeterminacy-membership function $\tilde{I}_A(x)$, and an interval falsity-membership function $\tilde{F}_A(x)$ that assigns a complex-valued membership grade to $\tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)$ for any $x \in \zeta$. The values of $\tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)$ and their sum may be any values within a unit circle in the complex plane and is therefore of the form

$$\tilde{T}_A(x) = [p_A^L(x), p_A^U(x)].e^{i[\mu_A^L(x),\ \mu_A^U(x)]}, \tag{24}$$

$$\tilde{I}_A(x) = [q_A^L(x), q_A^U(x)].e^{i[\nu_A^L(x),\ \nu_A^U(x)]} \tag{25}$$

$$\text{and } \tilde{F}_A(x) = [r_A^L(x), r_A^U(x)].e^{i[\omega_A^L(x),\ \omega_A^U(x)]} \tag{26}$$

All the amplitude and phase terms are real-valued and $p_A^L(x), p_A^U(x), q_A^L(x), q_A^U(x), r_A^L(x)$ and $r_A^U(x) \in [0,1]$, whereas $\mu_A(x), \nu_A(x), \omega_A(x) \in (0, 2\pi]$, such that the condition

$$0 \leq p_A^U(x) + q_A^U(x) + r_A^U(x) \leq 3 \tag{27}$$

is satisfied. An interval complex neutrosophic set $\tilde{A}$ can thus be represented in set form as:

$$\tilde{A} = \{\langle x, T_A(x) = a_T, I_A(x) = a_I, F_A(x) = a_F \rangle : x \in \zeta\}, \tag{28}$$

Where$T_A: \zeta. \rightarrow \{a_T: a_T \in C, |a_T| \leq 1\}, I_A: \zeta. \rightarrow \{a_I: a_I \in C, |a_I| \leq 1\}, F_A: \zeta. \rightarrow \{a_F: a_F \in C, |a_F| \leq 1\},$ and also $\left|T_A^U(x) + I_A^U(x) + F_A^U(x)\right| \leq 3.$ (29)

Definition 2.6 [16]

Let A and B be two ICNSs in ζ. The union, intersection and complement of two ICNSs are defined as:

The union of A and B denoted as $A \cup_N B$, is defined as:

$$A \cup_N B = \left\{\left(x, \tilde{T}_{A\cup B}(x), \tilde{I}_{A\cup B}(x), \tilde{F}_{A\cup B}(x)\right) : x \in X\right\},$$ (30)

Where, $\tilde{T}_{A\cup B}(x), \tilde{I}_{A\cup B}(x), \tilde{F}_{A\cup B}(x)$ are given by

$$T_{A\cup B}^L(x) = \left[(p_A^L(x) \vee p_B^L(x))\right].e^{j.\mu_{T_{A\cup B}}^L(x)},$$

$$T_{A\cup B}^U(x) = \left[(p_A^U(x) \vee p_B^U(x))\right].e^{j.\mu_{T_{A\cup B}}^U(x)}$$ (31)

$$I_{A\cup B}^L(x) = \left[(q_A^L(x) \wedge q_B^L(x))\right].e^{j.\mu_{I_{A\cup B}}^L(x)},$$

$$I_{A\cup B}^U(x) = \left[(q_A^U(x) \wedge q_B^U(x))\right].e^{j.\mu_{I_{A\cup B}}^U(x)},$$ (32)

$$F_{A\cup B}^L(x) = \left[(r_A^L(x) \wedge r_B^L(x))\right].e^{j.\mu_{F_{A\cup B}}^L(x)},$$

$$F_{A\cup B}^U(x) = \left[(r_A^U(x) \wedge r_B^U(x))\right].e^{j.\mu_{F_{A\cup B}}^U(x)}$$ (33)

The intersection of A and B denoted as $A \cap_N B$, is defined as:

$$A \cap_N B = \left\{\left(x, \tilde{T}_{A\cap B}(x), \tilde{I}_{A\cap B}(x), \tilde{F}_{A\cap B}(x)\right) : x \in X\right\},$$ (34)

Where, $\tilde{T}_{A\cap B}(x), \tilde{I}_{A\cap B}(x), \tilde{F}_{A\cap B}(x)$ are given by

$$T_{A\cap B}^L(x) = \left[(p_A^L(x) \wedge p_B^L(x))\right].e^{j.\mu_{T_{A\cup B}}^L(x)},$$

$$T_{A\cap B}^U(x) = \left[(p_A^U(x) \wedge p_B^U(x))\right].e^{j.\mu_{T_{A\cup B}}^U(x)}$$ (35)

$$I_{A\cap B}^L(x) = \left[(q_A^L(x) \vee q_B^L(x))\right].e^{j.\mu_{I_{A\cup B}}^L(x)},$$

$$I_{A\cap B}^U(x) = \left[(q_A^U(x) \vee q_B^U(x))\right].e^{j.\mu_{I_{A\cup B}}^U(x)},$$ (36)

$$F_{A\cap B}^L(x) = \left[(r_A^L(x) \vee r_B^L(x))\right].e^{j.\mu_{F_{A\cup B}}^L(x)},$$

$$F_{A\cap B}^U(x) = \left[(r_A^U(x) \vee r_B^U(x))\right].e^{j.\mu_{F_{A\cup B}}^U(x)}$$ (37)

The union and the intersection of the phase terms of the complex truth, falsity and indeterminacy membership functions can be calculated using any one of the following operations:

Sum:

$$\mu_{A\cup B}^{L}(x) = \mu_{A}^{L}(x) + \mu_{B}^{L}(x),$$

$$\mu_{A\cup B}^{U}(x) = \mu_{A}^{U}(x) + \mu_{B}^{U}(x), \tag{38}$$

$$v_{A\cup B}^{L}(x) = v_{A}^{L}(x) + v_{B}^{L}(x),$$

$$v_{A\cup B}^{U}(x) = v_{A}^{U}(x) + v_{B}^{U}(x), \tag{39}$$

$$\omega_{A\cup B}^{L}(x) = \omega_{A}^{L}(x) + \omega_{B}^{L}(x),$$

$$\omega_{A\cup B}^{U}(x) = \omega_{A}^{U}(x) + \omega_{B}^{U}(x), \tag{40}$$

Max:

$$\mu_{A\cup B}^{L}(x) = \max(\mu_{A}^{L}(x), \mu_{B}^{L}(x)),$$

$$\mu_{A\cup B}^{U}(x) = \max(\mu_{A}^{U}(x), \mu_{B}^{U}(x)), \tag{41}$$

$$v_{A\cup B}^{L}(x) = \max(v_{A}^{L}(x), v_{B}^{L}(x)),$$

$$v_{A\cup B}^{U}(x) = \max(v_{A}^{U}(x), v_{B}^{U}(x)), \tag{42}$$

$$\omega_{A\cup B}^{L}(x) = \max(\omega_{A}^{L}(x), \omega_{B}^{L}(x)),$$

$$\omega_{A\cup B}^{U}(x) = \max(\omega_{A}^{U}(x), \omega_{B}^{U}(x)), \tag{43}$$

Min:

$$\mu_{A\cup B}^{L}(x) = \min(\mu_{A}^{L}(x), \mu_{B}^{L}(x)),$$

$$\mu_{A\cup B}^{U}(x) = \min(\mu_{A}^{U}(x), \mu_{B}^{U}(x)), \tag{44}$$

$$v_{A\cup B}^{L}(x) = \min(v_{A}^{L}(x), v_{B}^{L}(x)),$$

$$v_{A\cup B}^{U}(x) = \min(v_{A}^{U}(x), v_{B}^{U}(x)), \tag{45}$$

$$\omega_{A\cup B}^{L}(x) = \min(\omega_{A}^{L}(x), \omega_{B}^{L}(x)),$$

$$\omega_{A\cup B}^{U}(x) = \min(\omega_{A}^{U}(x), \omega_{B}^{U}(x)), \tag{46}$$

"The game of winner, neutral, and loser":

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & if \quad p_A > p_B \\ \mu_B(x) & if \quad p_B > p_A \end{cases}, \tag{47}$$

$$v_{A\cup B}(x) = \begin{cases} v_A(x) & if \quad q_A < q_B \\ v_B(x) & if \quad q_B < q_A \end{cases}, \tag{48}$$

$$\omega_{A\cup B}(x) = \begin{cases} \omega_A(x) & if \quad r_A < r_B \\ \omega_B(x) & if \quad r_B < r_A \end{cases}. \tag{49}$$

Definition 2.7 [52]

Consider V be a non-void set. Two function are considered as follows:

$\rho=(\rho_T, \rho_I, \rho_F)$:V $\rightarrow$ $[\,0,1]^3$ and

$\omega=(\omega_T, \omega_I, \omega_F)$:VxV $\rightarrow$ $[\,0,1]^3$. We suppose

$$A= \{(\rho_T(x),\rho_T(y)) \mid \omega_T(x,y) \geq 0\}, \tag{50}$$

$$B= \{(\rho_I(x),\rho_I(y)) \mid \omega_I(x,y) \geq 0\}, \tag{51}$$

$$C= \{(\rho_F(x),\rho_F(y)) \mid \omega_F(x,y) \geq 0\}, \tag{52}$$

considered ω_T, ω_I and $\omega_F \geq 0$ for all set A,B, C since its is possible to have edge degree = 0 (for T, or I, or F).

The triad (V, ρ, ω) is defined to be complex neutrosophic graph of type 1 (CNG1) if there are functions

α:A$\rightarrow$ $[\,0,1]$, β:B$\rightarrow$ $[\,0,1]$ and δ:C$\rightarrow$ $[\,0,1]$ such that

$$\omega_T(x,y) = \alpha((\rho_T(x),\rho_T(y))) \tag{53}$$

$$\omega_I(x,y) = \beta((\rho_I(x),\rho_I(y))) \tag{54}$$

$$\omega_F(x,y) = \delta((\rho_F(x),\rho_F(y))) \text{ where x, y}\in \text{V.} \tag{55}$$

For each $\rho(x)= (\rho_T(x), \rho_I(x), \rho_F(x))$,,x$\in$ V are called the complex truth, complex indeterminacy and complex falsity-membership values, respectively, of the vertex x. likewise for each edge (x, y) : $\omega(x,y)=(\omega_T(x,y), \omega_I(x,y), \omega_F(x,y))$ are called the complex membership, complex indeterminacy membership and complex falsity values of the edge.

3 Interval Complex Neutrosophic Graph of Type 1

In this section, based on the concept of complex neutrosophic graph of type 1 [52], we define the concept of interval complex neutrosophic graph of type 1 as follows:

Definition 3.1.

Consider V be a non-void set. Two function are considered as follows:
$\rho=([\rho_T^L,\rho_T^U],[\rho_I^L,\rho_I^U], [\rho_F^L,\rho_F^U])$:V$\rightarrow$ $[\,0,1]^3$ and
$\omega=([\omega_T^L,\omega_T^U], [\omega_I^L,\omega_I^U], [\omega_F^L,\omega_F^U])$:VxV $\rightarrow$ $[\,0,1]^3$. We suppose
A= $\{([\rho_T^L(x),\rho_T^U(x)], [\rho_T^L(y),\rho_T^U(y)]) \mid \omega_T^L(x,y) \geq 0$
and $\omega_T^U(x,y) \geq 0 \}$, $\tag{56}$
B= $\{([\rho_I^L(x),\rho_I^U(x)], [\rho_I^L(y),\rho_I^U(y)]) \mid \omega_I^L(x,y) \geq 0$
and $\omega_I^U(x,y) \geq 0\}$, $\tag{57}$
C= $\{([\rho_F^L(x),\rho_F^U(x)], [\rho_F^L(y),\rho_F^U(y)]) \mid \omega_F^L(x,y) \geq 0$
and $\omega_F^U(x,y) \geq 0\}$, $\tag{58}$
We have considered ω_T, ω_I and $\omega_F \geq 0$ for all set A,B, C , since its is possible to have edge degree = 0 (for T, or I, or F).

The triad (V, ρ, ω) is defined to be an interval complex neutrosophic graph of type 1 (ICNG1) if there are functions

α:A$\rightarrow$ [0, 1] , β:B$\rightarrow$ [0, 1] and δ:C$\rightarrow$ [0, 1] such that

$\omega_T(x,y)=[\omega_T^L(x,y),$

$\omega_T^U(x,y)]= \alpha([\rho_T^L(x),\rho_T^U(x)],[\rho_T^L(y),\rho_T^U(y)])$ (59)

$\omega_I(x,y)=[\omega_I^L(x,y),$

$\omega_I^U(x,y)]= \beta([\rho_I^L(x),\rho_I^U(x)],[\rho_I^L(y),\rho_I^U(y)])$ (60)

$\omega_F(x,y)=[\omega_F^L(x,y),$

$\omega_F^U(x,y)]= \delta([\rho_F^L(x),\rho_F^U(x)],[\rho_F^L(y),\rho_F^U(y)])$ where x, y$\in$ V.   (61)

For each $\rho(x)=([\rho_T^L(x),\rho_T^U(x)], [\rho_I^L(x),\rho_I^U(x)],[\rho_F^L(x),\rho_F^U(x)]),x\in$ V are called the interval complex truth, interval complex indeterminacy and interval complex falsity-membership values, respectively, of the vertex x. likewise for each edge(x, y) :$\omega(x,y)=(\omega_T(x,y), \omega_I(x,y), \omega_F(x,y))$ are called the interval complex membership, interval complex indeterminacy membership and interval complex falsity values of the edge.

Example 3.2

Consider the vertex set be V={x, y, z, t} and edge set be E= {(x, y),(x, z),(x, t),(y, t)}

	x	y	z	t
$[\rho_T^L,\rho_T^U]$	$[0.5, 0.6]e^{j.\pi[0.8,0.9]}$	$[0.9 , 1]e^{j.\pi[0.7,0.8]}$	$[0.3, 0.4]e^{j.\pi[0.2,0.5]}$	$[0.8, 0.9]e^{j.\pi[0.1,0.3]}$
$[\rho_I^L,\rho_I^U]$	$[0.3, 0.4]e^{j.\pi[0.1,0.2]}$	$[0.2, 0.3]e^{j.\pi[0.5,0.6]}$	$[0.1, 0.2]e^{j.\pi[0.3,0.6]}$	$[0.5, 0.6]e^{j.\pi[0.2,0.8]}$
$[\rho_F^L,\rho_F^U]$	$[0.1, 0.2]e^{j.\pi[0.5,0.7]}$	$[0.6, 0.7]e^{j.\pi[0.2,0.3]}$	$[0.8, 0.9]e^{j.\pi[0.2,0.4]}$	$[0.4, 0.5]e^{j.\pi[0.3,0.7]}$

Table 1. Interval Complex truth-membership, indeterminacy-membership and falsity-membership of the vertex set.

Given the following functions

$\alpha(m,n)=[m_T^L(u) \vee n_T^L(u), m_T^U(u) \vee n_T^U(u)] . e^{j.\pi\mu_{A\cup B}(u)}$ (62)

$\beta(m,n)=[m_I^L(u) \wedge n_I^L(u), m_I^U(u) \wedge n_I^U(u)] . e^{j.\pi\nu_{A\cup B}(u)}$ (63)

$\delta(m,n)= [m_F^L(u) \wedge n_F^L(u), m_F^U(u) \wedge n_F^U(u)] . e^{j.\pi\omega_{A\cup B}(u)}$ (64)

Here,

A={$([0.5, 0.6]e^{j.\pi[0.8,0.9]}$, $[0.9, 1]e^{j.\pi[0.7,0.8]}$), $([0.5, 0.6]e^{j.\pi[0.8,0.9]}$, $[0.3, 0.4]e^{j.\pi[0.2,0.5]}$), $([0.5, 0.6]e^{j.\pi[0.8,0.9]}$, $[0.8, 0.9]e^{j.\pi[0.1,0.3]}$),$([0.9, 1.0]e^{j.\pi[0.7,0.8]}$, $[0.8, 0.9]e^{j.\pi[0.1,0.3]})$}

B={$([0.3, 0.4]e^{j.\pi[0.1,0.2]}$, $[0.2, 0.3]e^{j.\pi[0.5,0.6]}$), $([0.3, 0.4]e^{j.\pi[0.1,0.2]}$, $[0.1,$

$0.2]e^{j.\pi[0.3,0.6]}$), ([0.3, 0.4]$e^{j.\pi[0.1,0.2]}$, [0.5, 0.6]$e^{j.\pi[0.2,0.8]}$), ([0.2, 0.3]$e^{j.\pi[0.5,0.6]}$, [0.5, 0.6]$e^{j.\pi[0.2,0.8]}$)}

C={([0.1, 0.2]$e^{j.\pi[0.5,0.7]}$, [0.6, 0.7]$e^{j.\pi[0.2,0.3]}$), ([0.1, 0.2]$e^{j.\pi[0.5,0.7]}$, [0.8, 0.9]$e^{j.\pi[0.2,0.4]}$), ([0.1, 0.2]$e^{j.\pi[0.5,0.7]}$, [0.4, 0.5]$e^{j.\pi[0.3,0.7]}$), ([0.6, 0.7]$e^{j.\pi[0.2,0.3]}$, [0.4, 0.5]$e^{j.\pi[0.3,0.7]}$)}.

Then

ω	(x,y)	(x,z)	(x,t)	(y,t)
$[\omega_T^L,\omega_T^U]$	[0.9, 1]$e^{j.\pi[0.8,0.9]}$	[0.5, 0.6]$e^{j.\pi[0.8,0.9]}$	[0.8,0.9]$e^{j.\pi[0.8,0.9]}$	[0.9,1]$e^{j.\pi[0.8,0.9]}$
$[\omega_I^L,\omega_I^U]$	[0.2,0.3]$e^{j.\pi[0.5,0.6]}$	[0.1,0.2]$e^{j.\pi[0.3,0.6]}$	[0.3,0.4]$e^{j.\pi[0.2,0.8]}$	[0.2, 0.3]$e^{j.\pi[0.5,0.8]}$
$[\omega_F^L,\omega_F^U]$	[0.1, 0.2]$e^{j.\pi[0.5,0.7]}$	[0.1,0.2]$e^{j.\pi[0.5,0.7]}$	[0.1,0.2]$e^{j.\pi[0.5,0.7]}$	[0.4,0.5]$e^{j.\pi[0.5,0.7]}$

Table 2. Interval Complex truth-membership, indeterminacy-membership and falsity-membership of the edge set.

The figure 2 show the interval complex neutrosophic graph of type 1
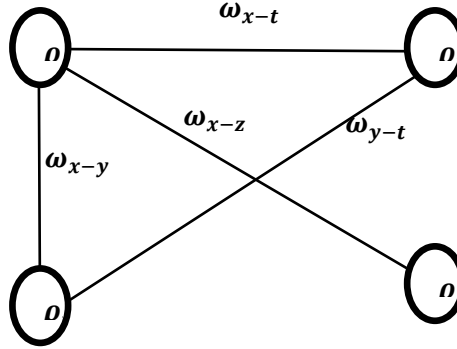
Fig 2. Interval complex neutrosophicgraph of type 1.

In classical graph theory, any graph can be represented by adjacency matrices, and incident matrices. In the following section ICNG1 is represented by adjacency matrix.

4 Representation of interval complex neutrosophic graph of Type 1 by adjacency matrix

In this section, interval truth-membership, interval indeterminate-membership and interval false-membership are considered independents. Based on the representation of complex neutrosophic graph of type 1 by adjacency matrix [52],

we propose a matrix representation of interval complex neutrosophic graph of type 1 as follow:

The interval complex neutrosophic graph (ICNG1) has one property that edge membership values (T, I, F) depends on the membership values (T, I, F) of adjacent vertices. Suppose $\xi=(V,\ \rho,\omega)$ is a ICNG1 where vertex set $V=\{v_1, v_2,\ldots,v_n\}$. The functions

α :A$\rightarrow$ (0, 1] is taken such that

$\omega_T^L(x,y) = \alpha((\rho_T^L(x),\rho_T^L(y)))$, $\omega_T^U(x,y) = \alpha((\rho_T^U(x),\rho_T^U(y)))$, where x, y$\in$ V and

A= $\{([\rho_T^L(x),\rho_T^U(x)], [\rho_T^L(y),\rho_T^U(y)]) | \omega_T^L$(x, y) ≥ 0 and ω_T^U(x, y) ≥ 0 \},

β :B$\rightarrow$ (0, 1] is taken such that

$\omega_I^L(x,y) = \beta((\rho_I^L(x),\rho_I^L(y)))$, $\omega_I^U(x,y) = \beta((\rho_I^U(x),\rho_I^U(y)))$, where x, y$\in$ V and

B= $\{([\rho_I^L(x),\rho_I^U(x)], [\rho_I^L(y),\rho_I^U(y)]) | \omega_I^L$(x, y) ≥ 0 and ω_I^U(x, y) ≥ 0 \}

and

δ :C$\rightarrow$ (0, 1] is taken such that

$\omega_F^L(x,y) = \delta((\rho_F^L(x),\rho_F^L(y)))$, $\omega_F^U(x,y) = \delta((\rho_F^U(x),\rho_F^U(y)))$, where x, y$\in$ V and

C= $\{([\rho_F^L(x),\rho_F^U(x)], [\rho_F^L(y),\rho_F^U(y)]) | \omega_F^L$(x, y) ≥ 0 and ω_F^U(x, y) ≥ 0 \}.

The ICNG1 can be represented by (n+1) x (n+1) matrix $M_{G_1}^{T,I,F}=[a^{T,I,F}(i, j)]$ as follows:

The interval complex truth membership (T), interval complex indeterminacy-membership (I) and the interval complex falsity-membership (F) values of the vertices are provided in the first row and first column. The (i+1, j+1)- th-entry are the interval complex truth membership (T), interval complex indeterminacy-membership (I) and the interval complex falsity-membership (F) values of the edge (x_i, x_j), i, j=1,…,n if i$\neq$j.

The (i, i)-th entry is $\rho(x_i)=(\rho_T(x_i),\ \rho_I(x_i),\ \rho_F(x_i))$, where i=1,2,…,n. the interval complex truth membership (T), interval complex indeterminacy-membership (I) and the interval complex falsity-membership (F) values of the edge can be computed easily using the functions α, β and δ which are in (1,1)-position of the matrix. The matrix representation of ICNG1, denoted by $M_{G_1}^{T,I,F}$, can be written as three matrix representation $M_{G_1}^T$, $M_{G_1}^I$ and $M_{G_1}^F$. For convenience representation $v_i(\rho_T(v_i)) = [\rho_T^L(v_i),\ \rho_T^U(v_i)]$, $v_i(\rho_I(v_i)) = [\rho_I^L(v_i),\ \rho_I^U(v_i)]$ and $v_i(\rho_F(v_i)) = [\rho_F^L(v_i),\ \rho_F^U(v_i)]$, for i=1, …., n

The $M_{G_1}^T$ can be therefore represented as follows

α	$v_1(\rho_T(v_1))$	$v_2(\rho_T(v_2))$	$v_n(\rho_T(v_n))$
$v_1(\rho_T(v_1))$	$[\rho_T^L(v_1), \rho_T^U(v_1)]$	$\alpha(\rho_T(v_1),\rho_T(v_2))$	$\alpha(\rho_T(v_1),\rho_T(v_n))$
$v_2(\rho_T(v_2))$	$\alpha(\rho_T(v_2),\rho_T(v_1))$	$[\rho_T^L(v_2), \rho_T^U(v_2)]$	$\alpha(\rho_T(v_2),\rho_T(v_2))$
…	….	…	…
$v_n(\rho_T(v_n))$	$\alpha(\rho_T(v_n),\rho_T(v_1))$	$\alpha(\rho_T(v_n),\rho_T(v_2))$	$[\rho_T^L(v_n), \rho_T^U(v_n)]$

Table 3. Matrix representation of T-ICNG1

The $M_{G_1}^I$ can be therefore represented as follows

β	$v_1(\rho_I(v_1))$	$v_2(\rho_I(v_2))$	$v_n(\rho_I(v_n))$
$v_1(\rho_I(v_1))$	$[\rho_I^L(v_1), \rho_I^U(v_1)]$	$\beta(\rho_I(v_1),\rho_I(v_2))$	$\beta(\rho_I(v_1),\rho_I(v_n))$
$v_2(\rho_I(v_2))$	$\beta(\rho_I(v_2),\rho_I(v_1))$	$[\rho_I^L(v_2), \rho_I^U(v_2)]$	$\beta(\rho_I(v_2),\rho_I(v_2))$
…	….	…	…
$v_n(\rho_I(v_n))$	$\beta(\rho_I(v_n),\rho_I(v_1))$	$\beta(\rho_T(v_n),\rho_I(v_2))$	$[\rho_I^L(v_n), \rho_I^U(v_n)]$

Table 4. Matrix representation of I-ICNG1

The $M_{G_1}^I$ can be therefore represented as follows

δ	$v_1(\rho_F(v_1))$	$v_2(\rho_F(v_2))$	$v_n(\rho_F(v_n))$
$v_1(\rho_F(v_1))$	$[\rho_F^L(v_1), \rho_F^U(v_1)]$	$\delta(\rho_F(v_1),\rho_F(v_2))$	$\delta(\rho_F(v_1),\rho_F(v_n))$
$v_2(\rho_F(v_2))$	$\delta(\rho_F(v_2),\rho_F(v_1))$	$[\rho_F^L(v_2), \rho_F^U(v_2)]$	$\delta(\rho_F(v_2),\rho_F(v_2))$
…	….	…	…
$v_n(\rho_F(v_n))$	$\delta(\rho_F(v_n),\rho_F(v_1))$	$\delta(\rho_F(v_n),\rho_F(v_2))$	$[\rho_F^L(v_n), \rho_F^U(v_n)]$

Table 5. Matrix representation of F-ICNG1

Here the Interval complex neutrosophic graph of first type (ICNG1) can be represented by the matrix representation depicted in table 9. The matrix representation can be written as three interval complex matrices one containing the entries as T, I, F (see table 6, 7 and 8).

$\alpha = \max(x, y)$	x([0.5, 0.6] $e^{j.\pi[0.8,0.9]}$)	y([0.9 , 1]. $e^{j.\pi[0.7,0.8]}$)	z([0.3, 0.4]. $e^{j.\pi[0.2,0.5]}$)	t([0.8, 0.9]. $e^{j.\pi[0.1,0.3]}$)
x([0.5, 0.6].$e^{j.\pi[0.8,0.9]}$)	[0.5, 0.6].$e^{j.\pi[0.8,0.9]}$	[0.9 , 1]. $e^{j.\pi[0.8,0.9]}$	[0.5, 0.6].$e^{j.\pi[0.8,0.9]}$	[0.8, 0.9]. $e^{j.\pi[0.8\ 0.9]}$
y([0.9 , 1]. $e^{j.\pi[0.7,0.8]}$)	[0.9 , 1]. $e^{j.\pi[0.8,0.9]}$	[0.9 , 1]. $e^{j.\pi[0.7,0.8]}$	**[0, 0]**	[0.9 , 1]. $e^{j.\pi[0.7,0.8]}$
z([0.3, 0.4]. $e^{j.\pi[0.2,0.5]}$)	[0.5, 0.6].$e^{j.\pi[0.8,0.9]}$	**[0, 0]**	[0.3, 0.4]. $e^{j.\pi[0.2,0.5]}$	**[0, 0]**
t([0.8, 0.9]. $e^{j.\pi[0.1,0.3]}$)	[0.8, 0.9]. $e^{j.\pi[0.8,0.9]}$	[0.9 , 1]. $e^{j.\pi[0.7,0.8]}$	**[0, 0]**	[0.8, 0.9]. $e^{j.\pi[0.1,0.3]}$

Table 6: Lower and upper Truth- matrix representationof ICNG1

$\beta = \min(x, y)$	x([0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$)	y([0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$)	z([0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$)	t([0.5, 0.6]. $e^{j.\pi[0.2,0.8]}$)
x([0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$)	[0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$	[0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$	[0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	[0.3, 0.4]. $e^{j.\pi[0.3,0.6]}$
y([0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$)	[0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$	[0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$	**[0, 0]**	[0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$
z([0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$)	[0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	**[0, 0]**	[0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	**[0, 0]**
t([0.5, 0.6]. $e^{j.\pi[0.2,0.8]}$)	[0.3, 0.4]. $e^{j.\pi[0.2,0.8]}$	[0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$	**[0, 0]**	[0.5 0.6]. $e^{j.\pi[0.2,0.8]}$

Table 7: Lower and upper Indeterminacy- matrix representation of ICNG1

$\delta = \min(x, y)$	x([0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$)	y([0.6, 0.7]. $e^{j.\pi[0.2,0.3]}$)	z([0.8, 0.9]. $e^{j.\pi[0.2,0.4]}$)	t([0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$)
x([0.1, 0.2].$e^{j.\pi[0.5,0.7]}$)	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$
y([0.6, 0.7].$e^{j.\pi[0.2,0.3]}$)	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.6, 0.7]. $e^{j.\pi[0.2,0.3]}$	**[0, 0]**	[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$
z([0.8, 0.9].$e^{j.\pi[0.2,0.4]}$)	[0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$	**[0, 0]**	[0.8, 0.9]. $e^{j.\pi[0.2,0.4]}$	**[0, 0]**
t[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$)	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$	**[0, 0]**	[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$

Table 8: Lower and upper Falsity- matrix representation of ICNG1

The matrix representation of ICNG1 can be represented as follows:

(α, β, δ)	X(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	y(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	z(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	t(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)
X(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.9, 0.1]. $e^{j.\pi[0.8,0.9]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$>	<[0.8, 0.9]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.2,0.8]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>
y(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.9, 0.1]. $e^{j.\pi[0.8,0.9]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	**<[0, 0], [0, 0], [0, 0]>**	<[0.9 1]. $e^{j.\pi[07,0.8]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$, [0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$>
z(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$>	**<[0, 0], [0, 0], [0, 0]>**	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	**<[0, 0], [0, 0], [0, 0]>**
t(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.8, 0.9]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.2,0.8]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.9 1]. $e^{j.\pi[07,0.8]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$, [0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$>	**<[0, 0], [0, 0], [0, 0]>**	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>

Table 9: Matrix representation of ICNG1.

Remark 1

If $\rho_T^L(x)$=$\rho_T^U(x)$,$\rho_I^L(x)$=$\rho_I^U(x)$=0 and $\rho_F^L(x)$=$\rho_F^U(x) = 0$ and the interval valued phase terms equals zero, the interval complex neutrosophic graphs type 1 is reduced to generalized fuzzy graphs type 1 (GFG1).

Remark 2

If $\rho_T^L(x)$=$\rho_T^U(x)$,$\rho_I^L(x)$=$\rho_I^U(x)$ and $\rho_F^L(x)$=$\rho_F^U(x)$and the interval valued phase terms equals zero, the interval complex neutrosophic graphs type 1 is reduced to generalized single valued graphs type 1 (GSVNG1).

Remark 3

If $\rho_T^L(x)$=$\rho_T^U(x)$,$\rho_I^L(x)$=$\rho_I^U(x)$ and $\rho_F^L(x)$=$\rho_F^U(x)$ the interval complex neutrosophic graphs type 1 is reduced to complex neutrosophic graphs type 1 (CNG1).

Remark 4

If $\rho_T^L(x) \neq \rho_T^U(x)$, $\rho_I^L(x) \neq \rho_I^U(x)$ and $\rho_F^L(x) \neq \rho_F^U(x)$ *and* the interval valued phase terms equals zero, the interval complex neutrosophic graphs type 1 is reduced to generalized interval valued graphs type 1 (GIVNG1).

Theorem 1

Given the $M_{G_1}^T$ be matrix representation of T-ICNG1, then the degree of vertex $D_T(x_k) = [\sum_{j=1, j \neq k}^n a_T^L(k+1, j+1), \sum_{j=1, j \neq k}^n a_T^U(k+1, j+1)], x_k \in V$

or

$$D_T(x_p) = [\sum_{i=1, i \neq p}^n a_T^L(i+1, p+1), \sum_{i=1, i \neq p}^n a_T^U(i+1, p+1) \, x_p \in V.$$

Proof

Similar to that of theorem 1 of [52].

Theorem 2

Given the $M_{G_1}^I$ be a matrix representation of I-ICNG1, then the degree of vertex $D_I(x_k) = [\sum_{j=1, j \neq k}^n a_I^L(k+1, j+1), \sum_{j=1, j \neq k}^n a_I^U(k+1, j+1)], x_k \in V$

or $\quad D_I(x_p) = [\sum_{i=1, i \neq p}^n a_I^L(i+1, p+1), \sum_{i=1, i \neq p}^n a_I^U(i+1, p+1)], x_p \in$ V.

Proof

Similar to that of theorem 1 of [52].

Theorem 3

Given the $M_{G_1}^F$ be a matrix representation of ICNG1, then the degree of vertex

$$D_F(x_k) = [\sum_{j=1, j \neq k}^n a_F^L(k+1, j+1), \sum_{j=1, j \neq k}^n a_F^U(k+1, j+1)], x_k \in V$$

or

$$D_F(x_p) = [\sum_{i=1, i \neq p}^n a_F^L(i+1, p+1), \sum_{i=1, i \neq p}^n a_F^U(i+1, p+1)], x_p \in V.$$

Proof

Similar to that of theorem 1 of [52].

Theorem 4

Given the $M_{G_1}^{T,I,F}$ be a matrix representation of ICNG1, then the degree of vertex $D(x_k) = (D_T(x_k), D_I(x_k), D_F(x_k))$ where

$$D_T(x_k) = [\sum_{j=1, j \neq k}^n a_T^L(k+1, j+1), \sum_{j=1, j \neq k}^n a_T^U(k+1, j+1)], x_k \in V.$$

$$D_I(x_k) ==[\sum_{j=1,j\neq k}^{n} a_I^L(k+1,j+1), \sum_{j=1,j\neq k}^{n} a_I^U(k+1,j+1)], x_k \in V.$$

$$D_F(x_k) ==[\sum_{j=1,j\neq k}^{n} a_F^L(k+1,j+1), \sum_{j=1,j\neq k}^{n} a_F^U(k+1,j+1)], x_k \in V.$$

Proof

The proof is obvious.

5 Conclusion

In this article, we have introduced the concept of interval complex neutrosophic graph of type1 as generalization of the concept of single valued neutrosophic graph type 1 (GSVNG1), interval valued neutrosophic graph type 1 (GIVNG1) and complex neutrosophic graph of type1(CNG1). Next, we processed to presented a matrix representation of it. In the future works, we plan to study some more properties and applications of ICNG type 1 define the concept of interval complex neutrosophic graphs type 2.

Acknowledgements

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John R. Edwards